Part 1b: The Minimal Complexity Algorithm of the Planck Scaffolding

Part 2 of a 3-part series on the Simulation Hypothesis

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Abstract

In Article 1a, a Planck-scale lattice was mapped to observed cosmological parameters, yielding estimates for the CMB temperature, Hubble constant and dark matter density. That macro-scale correlation presupposes a micro-scale architecture, which this supplement formalises as a minimal-complexity algorithm and implements as a reference program.

Treating the universe as a continuous internal evaluation function (the τ-loop) governed by a small set of structural axioms, we show that the discrete step and the primary units (Planck mass M, time T and momentum P) are forced by the algorithm's internal logic; that e and π arise as solutions of the minimal growth and oscillation laws the axioms specify, the latter with eigenvalue $\lambda_1 = \pi^2$; and that one further postulate fixes the expansion eigenvalue Ω. Requiring the unit-number exponents to be integers admits the family $(\theta_M, \theta_P, \theta_T) = (15, 16, -30)k$, of which $k = 1$ is minimal and the positive sign is forced by the monotonicity of the coordinate accumulation. The resulting render invariant $I = \pi^2\Omega^{15}$ is the volume of $SO(3)$ at radius Ω^5.

We then treat the branch domain, in which particles and photons are τ-loops running slowly against the scaffolding. A branch executing one loop over ψ steps has time-averaged mass m_P/ψ — the mass ratio is the slowdown ratio — and reproduces the electron's Compton observables from $\psi = 2^{20}3^3\pi^8\alpha^{-3}\Omega^{15}$ without additional parameters. Because one τ-loop advances the phase by π rather than 2π, the state returns only after two cycles; with the 3D domain being the hypersphere surface $S^3 \cong SU(2)$, which double-covers $SO(3)$, spin-$\frac{1}{2}$ becomes available without assuming a half-angle.

The τ is the engine driving the simulation via the pi component and the e component, both working as complementary 'rotors' (driving rotation and expansion). At the end of each cycle the τ 'deposits' 1 unit of Planck time (and the other Planck units). As the Planck units are by-products of τ rather than a substrate τ moves through, there is no independent time axis for the spiral to advance along,

1 Introduction: The Simulation Paradigm

The correlation between the Planck unit scaffolding and the Cosmic Microwave Background established in Article 1a [3] suggests that the universe may operate as a discrete computational process. In standard physics, spacetime and quantum fields are fundamental. In the simulation framework the discrete computational step is fundamental, and continuous or quantum-mechanical features are emergent properties of sufficiently many discrete accumulations.

To formalise this we require a minimal complexity algorithm. The universe cannot be pre-initialised with a "bag of constants"; primary units and constants must be generated on demand by the algorithm's own state machine. This supplement details the internal mechanics of a single step, showing that the primary physical units are structurally forced by the geometry of discrete counting. Familiarity with the main article [3] is assumed.

On the status of this text. The architecture described here is also implemented as a running program, and where the two disagree the program is the reference. Prose can be read two ways; code cannot. Function names are cited in the text as `like_this()`, referring to `spiral_branch.py`, there are also 2 accompanying animation programs; `tau_animation.py` and `branch_animation.py`.

1.1 Axioms of the Minimal Complexity Algorithm

The derivations in this paper rest on two tiers of assumption, kept separate because they carry different epistemic weight.

Counters. Two counters suffice:

symbol	range	behaviour
n	$1, \ldots, B$	internal τ-cycle counter; **resets** at $n = B$
t	$1, 2, 3, \ldots$	spiral step; increments when n completes B; **never resets**

t *is* the number of 'physical' steps (each step deposits a set of Planck units). B = 15.

Tier 1: Foundational axioms. These define the minimal computational framework. No *empirical* constant appears in Tier 1; the only numbers are structural integers and the mathematical consequences of the postulated geometry.

Axiom 1 (Continuous Simulation Engine)**.** The universe evolves via a continuous internal evaluation function (the τ-loop). The discrete step is not an externally imposed clock; it emerges as a collapse event when the τ-loop satisfies its boundary condition.

Axiom 2 (Orthogonal Collapse)**.** Once per spiral step t — that is, when the internal counter n completes B cycles, not on each internal cycle — the lattice deposits exactly one unit of real coordinate accumulation onto the running sum $|z_t|^2$ (the coordinate/mass domain) and generates a strictly orthogonal imaginary phase rotation (the radiation domain).

Structural identification. Because the deposit is exactly one unit and t never resets, the running sum telescopes exactly, with no asymptotic correction at any t:

$$|z_t|^2 = t, \qquad |z_t| = \sqrt{t}. \tag{1}$$

Geometrically this is the *primal box*: the square erected on the spiral radius has side $\sqrt{t}$ and area t, so each completed step adds exactly one unit box. The side is the wave-state quantity (a square root, sign- and phase-carrying); the area is the point-state quantity (a positive integer). This axiom fixes only the *modulus* $|z_t|$; the phase of z_t is set independently by Axiom 3. In particular a complex radius is permitted — $r_t^2 = t$ would force the phase into $\{0, \pi\}$, but the derived condition is $|z_t|^2 = t$, which admits any phase.
Remark (open loops deposit nothing, and branches deposit nothing). The deposit is conditioned on loop *completion.* A τ-loop that remains open accumulates phase and contributes no box until it closes: a never-closing loop (photon) deposits nothing at any t. A branch is a single τ-loop running ψ times slower than the scaffolding loop (Section 5); it has therefore not completed its first Ω^B and generates no new Planck units at all. At closure it *marks* an existing box rather than depositing a new one, so this axiom's one-unit rule is not violated by branch closure — branch closure is not a scaffolding collapse.

Axiom 3 (Unit Action / Momentum Asymptote)**.** The product of the coordinate amplitude and the phase rate approaches unity as the spiral accumulates steps: $|z_t|\,\Delta\Theta_t \to 1$.

Structural identification. With $|z_t| = \sqrt{t}$ this fixes the phase increment as $\Delta\Theta_t \to 1/\sqrt{t}$, and together with Axiom 2 it forces the Theodorus step operator $S_t = 1 + i/\sqrt{t}$ (Section 3.1) — a derived structural consequence, not a further postulate. Note that the conserved quantity is $r\omega \to 1$, a constant *tangential speed*; it is not angular momentum, for which $r^2\omega$ would have to be constant and in fact diverges.

Axiom 4 (Minimal Self-Referential Growth)**.** Within one τ-cycle the loop evaluates according to the simplest self-referential growth law: its rate of change is proportional to its own current value, with unit proportionality, and its internal parameter s is calibrated one-to-one with the cycle:

$$\frac{d\tau}{ds} = \tau, \qquad \tau(0) = 1, \qquad \Delta s = 1. \tag{2}$$

The initial condition $\tau(0) = 1$ holds at the start of *every* cycle, not only the first: τ and s both reset at each cycle boundary, so each cycle is a fresh evaluation of the same law rather than a continuation of the previous value.

Structural identification. The equation is the defining generator of the exponential; its solution is $\tau(s) = e^s$, so $\tau(1) = e$. Thus e is not derived from more primitive axioms — it is the natural coordinate of the logarithmic action $\Delta(\ln\tau) = 1$ imposed by the calibration. The reset is a selected convention, not a theorem: the bare differential equation is equally consistent with a τ that accumulates. It is however the reading under which Axiom 8's closure condition holds at every cycle rather than only the first.

Axiom 5 (Minimal Bounded Oscillation)**.** The phase-generating oscillation internal to the τ-loop obeys the simplest linear equation admitting bounded, non-monotonic behaviour in the logarithmic coordinate $u = \ln\tau$:

$$w'' + \lambda w = 0, \qquad w(0) = w(1) = 0, \tag{3}$$

on $u \in [0,1]$ (i.e. $\tau \in [1,e]$, the image of one cycle under Axiom 4).

Structural identification (eigenvalue). The solution vanishing at $u = 0$ is $w = A\sin(\sqrt{\lambda}\,u)$; imposing $w(1) = 0$ forces $\sqrt{\lambda} = k\pi$. The lowest mode is

$$\lambda_1 = \pi^2, \qquad w_1(u) = \sin(\pi u), \qquad \text{i.e.} \qquad w(\tau) = \sin(\pi\ln\tau). \tag{4}$$

The *eigenvalue* is $\lambda_1 = \pi^2$; $\pi = \sqrt{\lambda_1}$ is the derived *wavenumber.* This distinction matters downstream: the render invariant's π^2 is λ_1 carried through intact, not a secondary quantity squared.
Structural identification (the half-cycle). The phase accumulated over one complete loop is

$$\varphi_{\text{loop}} = \pi\ln\tau\Big|_{\tau=1}^{\tau=e} = \pi, \qquad \text{not } 2\pi. \tag{5}$$

One τ-loop is a *half* cycle. Consequently the complex wave-state acquires a factor $e^{i\pi} = -1$ per loop, and the collapse state is $-\Omega^2$ rather than $+\Omega^2$ (Section D). Two loops are required to return to $+\Omega^2$. This is the origin of the phase convention used throughout, of the collapse sign, and — via the 4π periodicity it induces — of spin-$\frac{1}{2}$ in Section 5.

Axiom 6 (Tick Hierarchy)**.** The internal counter n indexes completion of one τ-cycle (Axioms 4–5). A cycle is not by itself a physically observable Planck step. Identifying n with the grading counter of Section 4.2, the integer-lock condition is first satisfied at $n = B$, where B is the minimal positive integer forced by that lock ($B = 15$, Section 4.2). At that point n resets to 1, the spiral step t increments by 1, and one primal box is deposited (Axiom 2). For $1 \le n < B$ the lattice is in a sub-Planckian bootstrapping phase: cycles accumulate without yet constituting an integer-graded Planck event.

Structural identification (synchronisation). Because t advances only in whole units and there is no clock external to the lattice, any process that closes onto the spiral must close on a spiral step. This is a genuine constraint, not a convenience: it requires the closure count of any branch to be an exact integer number of steps (Section 5), and it is what makes t a single global index — "now" is one value for the whole lattice, which is the sense in which the scaffolding is uniform everywhere.

Tier 2: Structural postulates. Axioms 1–6 establish the discrete recursion, the domain duality, the tick hierarchy, and the identification of e and π as the natural coordinates of the τ-loop. They do not fix a unique way to combine π and e into a single expansion eigenvalue

$$\Omega = \sqrt{\pi^e e^{(1-e)}} = 2.007\,134\,9543\ldots \tag{6}$$

The following closes that gap. Both the functional form and the point at which it is evaluated are postulated, not derived.

Axiom 7 (Capacity Functional Postulate)**.** To link the spatial phase boundary (π) with the loop boundary (e), the simulation postulates

$$F(x) = e\left(\frac{\pi}{x}\right)^x, \tag{7}$$

and postulates that it is evaluated at the natural loop boundary $x = e$ rather than at F's own extremum $x = \pi/e$ (where $F(\pi/e) \approx 8.63$ against $F(e) \approx 4.03$). The expansion eigenvalue is $\Omega^2 := F(e)$.

Axiom 8 (Emergent Discrete Step)**.** The simulation evaluates the wave-state amplitude $A(\tau) = \Omega^{\ln \tau + 1}$. A τ-cycle completes when the loop satisfies its boundary condition (Axioms 4–5), at which point the amplitude has matured to Ω^2 and the state collapses.

Structural identification (gain versus endpoint). $A(1) = \Omega^1$ and $A(e) = \Omega^2$, so the *gain* per cycle is Ω, while Ω^2 is the endpoint *value*. Over the B cycles of one spiral step the accumulated gain is therefore

$$\underbrace{\Omega \times \Omega \times \cdots \times \Omega}_{B} = \Omega^B = \Omega^{15}, \tag{8}$$

not Ω^{2B}. Combined with (5), the full complex state at collapse is $\Psi(e) = \Omega^2 e^{i\pi} = -\Omega^2$.

Axiom 9 (Two-Input Principle)**.** The simulation requires exactly two *empirical* external inputs: the fine-structure constant α and the dimensionless anchor $f(y)$, which fixes the absolute numerical scale [4].

Beyond these, the architecture requires the following *structural postulates*, recorded explicitly rather than hidden:

1. the continuous τ-loop engine (Axioms 1, 4, 5), which identifies e and π;
2. the capacity functional and its evaluation point (Axiom 7), which fixes Ω;
3. the integer-lock principle (Section 4.2), which requires unit-number exponents to be integers and selects $B = 15$.

Not counted, because derived rather than postulated: the Theodorus step operator (Section 3.1, from Axioms 2–3). All other numerical values (c, $\hbar$, G, m_P, t_P) are computed on demand. The model therefore requires **two empirical inputs and three structural postulates**.

Remark (on computing e and π). These are computable from the model's own primitives by series summation, requiring no external lookup, unlike α. A Basel summation ($\sum k^{-2} \to \pi^2/6$) for example may evolve with the spiral steps t providing the integers.

$$\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots. \tag{9}$$

2 The Minimal Complexity Algorithm

We model the universe as a state machine driven by a `WHILE` loop. The physical clock advances only when the internal evaluation (the τ-loop) completes and the state collapses. The architecture consists of two coupled fibers:

- **Phase-amplitude fiber** (Axioms 1, 4, 5): a continuous logarithmic spiral in the complex plane, governed by e and π, whose amplitude matures to Ω^2 while its phase advances by π — a half cycle, so the state collapses to $-\Omega^2$.
- **Coordinate fiber** (Axioms 2, 3): a discrete algebraic spiral (Theodorus), governed by orthogonal right-triangle accumulation, depositing one unit per spiral step t.

These fibers are not derivable from one another; they are orthogonal aspects of the same collapse event, coupled through the counters and the render invariant $\pi^2\Omega^{15}$.

2.1 Domain partition

The following distinction organises the remainder of the paper and determines which methods transfer between sections:

	spiral domain	**branch domain**
state	point-state	wave-state
coordinates	has them	none
α	absent (M, T, P, V, L)	present
closure	every step	after ψ steps, or never

A corollary worth stating, because it prevents a recurring confusion: a *spiral-domain description of a branch object is a projection, not a definition.* Descriptions of the wave-state phrased in point-state vocabulary — "no fixed coordinates", "a probability density" — are what the spiral domain is obliged to say about something it cannot represent, not statements about the object itself. The resemblance to the standard quantum-mechanical account is therefore expected rather than coincidental.

2.2 The simulation loop

Listing 1: The Universe Simulation Loop

```
t = 0;                    // spiral step; NEVER resets. t IS the Planck step.
n = 1;                    // internal tau-cycle counter; resets at B.
B = 15;                   // minimal integer base (Section, integer lock)

// The Omega equation is hardcoded into the simulation source-code
hardcoded_Omega_equation = sqrt(pi^e * e^(1-e));

WHILE (simulation_active) {

    // 1. RUN ONE TAU-CYCLE
    // pi and e are computed from the model's own primitives by series
    // summation. WHICH series is an open structural choice; see the
    // remark following the Two-Input Principle.
    pi_n = update_series_pi(t);
    e_n  = update_series_e(t);
    Omega_n = sqrt( (pi_n)^(e_n) * e_n^(1 - e_n) );
```

```
    // tau RESETS to 1 at the start of every cycle (Axiom, growth law)
    tau = 1;
    WHILE (tau < e) {
        // amplitude  A = Omega^(ln tau + 1):  Omega  ->  Omega^2
        // phase      phi = pi ln tau       :  0      ->  pi   (HALF cycle)
        expand_spiral(Omega_n, tau);
        rotate_spiral(Omega_n, tau);
        tau = tau + delta_tau;
    }

    // 2. CYCLE BOUNDARY
    // amplitude has matured to Omega^2 and the phase factor is e^{i pi},
    // so the collapse state is -Omega^2. The GAIN this cycle is Omega,
    // not Omega^2; B cycles therefore accumulate Omega^B = Omega^15.
    collapse_wave_state();
    n = n + 1;

    // 3. RENDER EVENT: when the internal counter completes B cycles
    if (n == B) {
        n = 0;                     // internal counter resets
        t = t + 1;                 // spiral step advances
        deposit_primal_box();      // |z_t|^2 = t exactly
        render_physical_units();  // unpack pi^2 Omega^15 into P, M, T

    // 4. DERIVED QUANTITIES computed on demand from M, T, P and alpha.
    //    Internal scale is fixed by the dimensionless anchor f(y).
    compute_derived_quantities();
    }
}
```

3 The Primary Units: From Structural Postulates to the Render Event

In Article 1a, the mass/length domain scales linearly with the step index t, while the radiation domain scales as $\sqrt{t}$. This domain duality is not imposed on the algorithm; it is a mathematical consequence of the Theodorus step operator.

3.1 Derivation of the Theodorus Step Operator

The geometry of the Spiral of Theodorus is not an arbitrary starting assumption but a forced consequence of Axioms 2 and 3 alone. Its content is independent of the τ-loop (Axioms 4–5); their coupling in time, rather than content, is made precise in Section 3.7.

By Axiom 2, each step adds exactly one unit of real coordinate and an orthogonal imaginary phase: if the squared coordinate amplitude is t, the real increment advances it to $t+1$. By Axiom 3, the phase rate is the reciprocal of the coordinate amplitude in the mature limit; as the amplitude is $\sqrt{t}$, the phase increment must be $1/\sqrt{t}$. For small angles the tangent of the increment equals the ratio of imaginary to real components, and since the real component is 1, the unique operator satisfying both conditions is

$$S_t = 1 + \frac{i}{\sqrt{t}}. \tag{10}$$

The recursion $z_{t+1} = z_t S_t$ (with $z_1 = 1$) is therefore a derived theorem, not an independent postulate. Its modulus is

$$|S_t| = \sqrt{1 + \frac{1}{t}} = \sqrt{\frac{t+1}{t}}, \tag{11}$$

so by induction the squared amplitude telescopes exactly to $|z_t|^2 = t$. The real component advances $|z_t|^2$ by exactly one integer per step, and $M = 1$ is the read-out of that real projection.

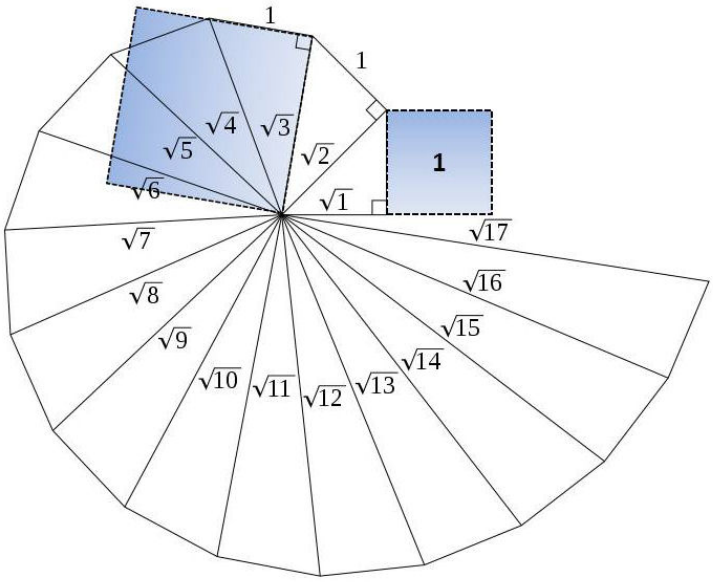

Figure 1: The Theodorus spiral lattice geometry (from Wikipedia).

3.2 The Phase Increment and Domain Duality

The argument of S_t is $\arctan(1/\sqrt{t})$, so the accumulated phase after t steps is

$$\Theta_t = \sum_{k=1}^{t-1} \arctan\left(\frac{1}{\sqrt{k}}\right) \longrightarrow 2\sqrt{t} + K, \qquad K = -2.157782997\dots \tag{12}$$

the asymptotic form following from Euler–Maclaurin summation, with K the Theodorus constant. The phase grows as $\sqrt{t}$, not as t: it belongs to a different scaling class from the squared amplitude. Separating the operator,

$$S_t = \underbrace{1}_{\text{coordinate-defining}} + \underbrace{\frac{i}{\sqrt{t}}}_{\text{phase-generating}}, \tag{13}$$

the real part defines coordinates ($|z_t|^2 = t$, integer), governing mass, volume and macroscopic time; the imaginary part generates phase ($\Theta_t \approx 2\sqrt{t}$, irrational), governing wavelength, frequency and temperature [3]. At this level the imaginary component has no spatial coordinate to correlate with: an independent spatial manifold does not yet exist.

3.3 Symmetry Conservation and the Render Event

The τ-loop (logarithmic, transcendental) and the Theodorus spiral (algebraic, discrete) are geometrically incompatible as descriptions of a single curve. They are orthogonal fibers of the simulation state, coupled at the render event.

During the sub-Planckian sequence ($n = 1$ to $B - 1$) the τ-loop accumulates a dimensionless invariant

$$I = \pi^2 \Omega^{15} = 341152.343709\dots \tag{14}$$

conserved in the sense that it may be repackaged without altering its value. The quantities P, M, T are not independently derived here; they satisfy the invariant subject to two constraints: the phase-balance equation $P^{15}T^2/M^{12} = I$, and integrality of the unit-number exponents $\theta(\cdot)$.

The parameterisation (introduced in Article 6. Planck unit geometries)

$$P = \Omega r^2, \qquad M = \frac{r^4}{v}, \qquad T = \pi\frac{r^9}{v^6} \tag{15}$$

carries the assignment of Ω and π content to P, M and T. Of the quantities here, only scalars r and v constitute a genuine *gauge choice* — bookkeeping variables rescalable without altering any prediction — unlike the Ω, π assignment, which is fixed. The render event is therefore a threshold crossing rather than a symmetry-breaking process: I, well-defined throughout the sub-Planckian phase, is unpacked into physical containers once the integer lock is first satisfied.

3.3.1 The invariant is a group volume

The invariant admits an exact geometric reading. Since $\Omega^{15} = (\Omega^5)^3$, and $\mathrm{Vol}(\mathbb{RP}^3)$ at radius a is $\pi^2 a^3$,

$$I = \pi^2\big(\Omega^5\big)^3 = \mathrm{Vol}\big(\mathbb{RP}^3\big) = \mathrm{Vol}\big(SO(3)\big) \quad \text{at radius } \Omega^5, \tag{16}$$

while the double cover has exactly twice that volume,

$$\mathrm{Vol}\big(S^3\big) = \mathrm{Vol}\big(SU(2)\big) = 2I. \tag{17}$$

The render lands on the volume of the *rotation group*; the full state space is its double cover. Two consequences are used later. Ω^5 is identified as the monopole quantum (Section 5), so the radius in (16) is not an arbitrary length. And the factor 2 between (16) and (17) is the same factor that appears as the τ-loop's half cycle, as the distinction between the observable (2π-periodic) and the configuration (4π-periodic), and as mass seeing ψ while spin sees 2ψ. These are not four coincidences but one double cover seen four ways, and they are the origin of spin-$\frac{1}{2}$ in Section 5.

3.3.2 The unit number is fixed by the scalars

The grading θ is not independent information. Writing each quantity's scalar content as $r^a v^b$,

$$\theta = 8a + 17b, \tag{18}$$

with r, v carrying unit numbers 8 and 17. This reproduces every entry of Table 1:

	scalars	$8a+17b$	θ		scalars	$8a+17b$	θ
M	r^4v^{-1}	15	15	A	$r^{-6}v^3$	3	3
T	r^9v^{-6}	−30	−30	K	$r^{-6}v^4$	20	20
P	r^2	16	16	σ_e	r^3v^{-2}	−10	−10
V	v	17	17	μ_0	r^7	56	56
L	r^9v^{-5}	−13	−13	h	$r^{13}v^{-5}$	19	19

Consistency is internal: $\theta(V) = 17$ because V's scalar is v, and $\theta(P) = 16 = 2 \times 8$ because P's scalar is r^2. A practical consequence, easy to overcount: the unit-number check and the scalar check are *the same check*. A construction satisfying one satisfies the other automatically.

3.4 Emergence of e, π, and Ω

3.4.1 Identification of e: the loop boundary

One unit of logarithmic action is required within the τ-loop. Since $A(\tau) = \Omega^{\ln\tau+1}$, the change in Ω-exponent is $\Delta k = \ln\tau_{\text{final}} = 1$, whose unique solution is $\tau_{\text{final}} = e$. Euler's number is the natural coordinate of the logarithmic action imposed by Axiom 4, not an arbitrary choice.

3.4.2 Identification of π: the boundary phase

The phase-generating oscillation is a standing wave in $u = \ln \tau$,

$$w(\tau) = \sin(\pi \ln \tau), \tag{19}$$

vanishing at both endpoints of $u \in [0, 1]$. The phase accumulated over the full loop is

$$\varphi_{\text{boundary}} = \pi \ln e = \pi, \tag{20}$$

and the Planck time unit $T = \pi$ counts this *half* cycle: one loop advances the phase by π, not 2π, so the complex state acquires a factor $e^{i\pi} = -1$ and collapses to $-\Omega^2$ (Section D). Two loops are required to return.

The *eigenvalue* of the boundary-value problem is $\lambda_1 = \pi^2$; $\pi = \sqrt{\lambda_1}$ is the derived *wavenumber*. The distinction is not cosmetic: the invariant's π^2 is λ_1 carried through intact, not a secondary quantity squared.

As noted, both π and e are computable from the model's own primitives by series summation, with precision increasing as the lattice ages.

3.4.3 Definition of Ω via the capacity functional

Following Axiom 7,

$$F(x) = e\left(\frac{\pi}{x}\right)^x = \frac{e\pi^x}{x^x}, \tag{21}$$

which measures how far the spatial (π-based) accumulation exceeds the self-referential (x^x) accumulation. F is governed by two competing points: its own maximum at $x_{\text{geom}} = \pi/e \approx 1.156$, and the natural loop boundary $x_{\text{nat}} = e$. Evaluating at the boundary,

$$\Omega^2 = F(e) = \pi^e e^{1-e}, \qquad \Omega = \sqrt{\pi^e e^{1-e}} \approx 2.0071349543, \tag{22}$$

taking the positive root since the lattice expands outward. That the functional is evaluated at $x = e$ rather than at its own extremum is part of the postulate, not a derivation.

The τ-loop then evaluates $A(\tau) = \Omega^{\ln \tau + 1}$, spanning $\Delta \ln \tau = 1$ and terminating at $\tau = e$, where $A(e) = \Omega^2$. The *gain* per cycle is Ω, so B cycles accumulate $\Omega^B = \Omega^{15}$, not Ω^{2B}.

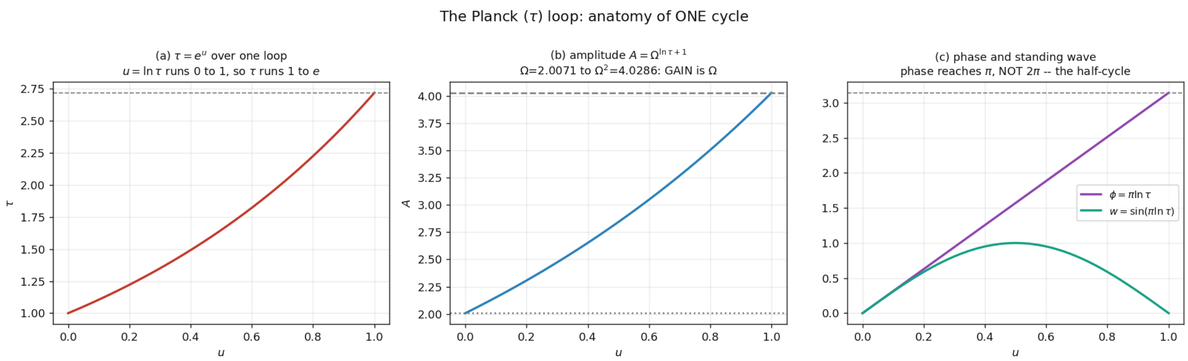

Figure 2: see *spiral branch.py*, *planck tau.png*

3.5 The Momentum Invariant in the Continuum Limit

In the continuum limit the amplitude is $\sqrt{t}$ and the phase $2\sqrt{t}$, so

$$\text{amplitude} \times \frac{d}{dt}(\text{phase}) = \sqrt{t} \cdot \frac{1}{\sqrt{t}} = 1. \tag{23}$$

This product locks to unity for all t: Planck momentum is an emergent, scale-invariant property of the mature lattice. Note that the conserved quantity is $r\omega \to 1$, a constant tangential speed, and not angular momentum, for which $r^2\omega$ would have to be constant and in fact diverges.

3.6 The Quantization of Action

Define the step action a_t as the product of coordinate amplitude and phase increment:

$$a_t = R_t\,\Delta\Theta_t = \sqrt{t}\,\arctan\!\left(\frac{1}{\sqrt{t}}\right) = 1 - \frac{1}{3t} + O(t^{-2}). \tag{24}$$

The action per step approaches exactly 1 as the lattice matures: the quantum of action $\hbar$ is an asymptotic property of the expanding spiral, not a primordial axiom.

3.7 Clock and Counter: The Disk Construction

Sections 3.1–3.4 establish the τ-loop and the spiral as separate structures governed by disjoint equations: $S_t = 1 + i/\sqrt{t}$ references only t, while $d\tau/ds = \tau$ and $w'' + \lambda w = 0$ reference only s, τ. The relationship is asymmetric: Axiom 8 is explicit that the τ-loop's boundary condition, and nothing else, advances the counter. The Theodorus recursion has no completion condition of its own; it describes what is deposited at a step, not when one occurs. The τ-loop is the pacemaker and the spiral is read out at each closure — content-independent, but not time-independent.

3.7.1 The disk construction

Rather than the recursion $z_{t+1} = z_t S_t$, consider the two components directly:

$$r_t := \sqrt{t}, \tag{25}$$

$$\Theta_t := \sum_{k=1}^{t-1} \arctan\!\left(\frac{1}{\sqrt{k}}\right). \tag{26}$$

Neither requires the other, and neither requires anything from τ beyond the bare fact that a step has occurred.

Lemma 1 (Polar reformulation)**.** $z_t = r_t e^{i\Theta_t}$ for all $t \geq 1$, where z_t is generated by $z_{t+1} = z_t S_t$, $z_1 = 1$.

Proof. By induction. Base: $r_1 = 1$, $\Theta_1 = 0$, so $z_1 = 1$. Step: assume $z_t = r_t e^{i\Theta_t}$. Then $|z_{t+1}| = r_t\sqrt{1 + 1/t} = \sqrt{t+1} = r_{t+1}$ and $\arg(z_{t+1}) = \Theta_t + \arctan(1/\sqrt{t}) = \Theta_{t+1}$. □

A (mathematical) disk construction is not an approximation of the recursive one; it is the same sequence of points, generated by two independent quantities rather than one compounding multiplication and rotating in its own right. (`spiral_radius()`, `spiral_theta()`).

3.7.2 Pitch

Pitch. Since $\Theta_t \to 2r_t$, one full turn ($\Delta\Theta = 2\pi$) corresponds to a radial advance

$$\Delta r = \pi. \tag{27}$$

The spiral is asymptotically Archimedean with pitch exactly π: successive turns lie π Planck lengths apart. Convergence is slow — the implied pitch is 8% above π at $t = 100$ and 0.3% at $t = 10^5$, converging as $t^{-1/2}$ — so early configurations are pre-asymptotic and cannot use this value.

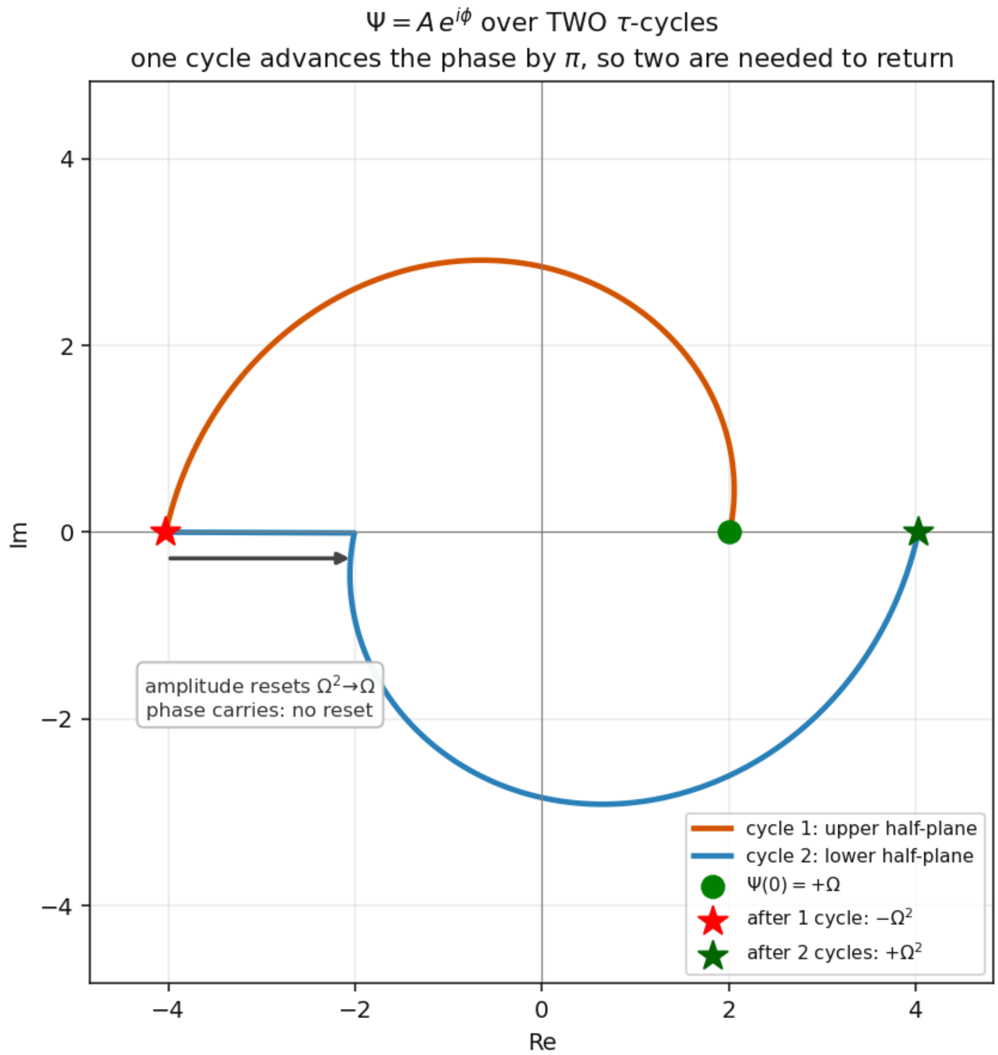

Figure 3: see *spiral branch.py*, *two cycle return.png*

4 Physical Constants from Ω and α

4.1 The Base-15 Phase Closure

While one τ-cycle generates an Ω gain, the physical universe requires simultaneous alignment of multiple dimensional phases. As Table 1 shows, a minimum of $B = 15$ cycles is required to construct the Planck units, so the master scalar for one complete structure is $\pi^2\Omega^{15}$:

$$\frac{P^{15}T^2}{M^{12}} = \pi^2\Omega^{15}. \tag{28}$$

Substituting (15),

$$\frac{P^{15}T^2}{M^{12}} = \frac{(\Omega r^2)^{15}(\pi r^9/v^6)^2}{(r^4/v)^{12}} = \pi^2\Omega^{15},$$

with r and v cancelling completely. This confirms that $\pi^2\Omega^{15}$ is the master scalar in the sense that matters here: independent of the internal r, v gauge. It is a separate and stronger claim that a corresponding relationship holds among the physical (SI) units themselves, and that claim is *not* established by the cancellation above; it is the subject of the statistical analysis in Article 6 [9].

Attribute	Quantity	Object	Scalar	u^θ	unit
mass	M	(1)	$\frac{r^4}{v}$	u^{15}	kg
time	T	(π)	$\frac{r^9}{v^6}$	u^{-30}	s
sqrt momentum	P	(Ω)	r^2	u^{16}	$\sqrt{\frac{\mathrm{kg\,m}}{\mathrm{s}}}$
velocity	V	$\frac{2\pi P^2}{M} = (2\pi\Omega^2)$	v	u^{17}	m/s
length	L	$VT = (2\pi^2\Omega^2)$	$\frac{r^9}{v^5}$	u^{-13}	m
current	A	$\frac{2^4V^3\alpha}{P^3} = (2^7\pi^3\alpha\Omega^3)$	$\frac{v^3}{r^6}$	u^3	A
temperature	K	$\frac{AV}{2\pi} = (2^7\pi^3\alpha\Omega^5)$	$\frac{v^4}{r^6}$	u^{20}	K

Table 1: The θ column is the unit-number invariant, and equals $8a + 17b$ for scalar content $r^a v^b$ (Eq. 18).

4.2 Diophantine System with Constraints

Start from

$$\begin{cases} 15\theta_P + 2\theta_T - 12\theta_M = 0 \\ 2\theta_M + \theta_T = 0 \end{cases} \qquad \theta_P, \theta_T, \theta_M \in \mathbb{Z} \setminus \{0\}.$$

The second gives $\theta_T = -2\theta_M$; substituting,

$$15\theta_P - 16\theta_M = 0.$$

Since $\gcd(15, 16) = 1$, every integer solution is

$$\boxed{\theta_P = 16k, \quad \theta_M = 15k, \quad \theta_T = -30k, \qquad k \in \mathbb{Z} \setminus \{0\}}. \tag{29}$$

Pairwise distinctness holds automatically for every non-zero k, since 16, 15 and -30 differ.

During the B sub-Planckian cycles the lattice operates purely on dimensionless constants, scaling until the container reaches the exact value $\pi^2\Omega^{15}$. At completion a substitution is permitted: because P, M, T relate to the boundary conditions via (28), the system undergoes an algebraic exchange. The dimensionless value is strictly conserved but its format is exchanged — the pure mathematics of π and Ω translated into physical dimensions. This can happen *only* at this junction: although the τ-loop has access to 1 and π throughout, it must wait for P^{15} before the 1 can be transferred to M and the π to T.

The macroscopic universe does not slowly grow from the sub-Planckian scale; it renders by substitution when the τ-loop completes its construction of $\pi^2\Omega^{15}$. Observers within the simulation perceive a physical mass-space-time universe while the sum remains net-zero and dimensionless.

4.2.1 The general solution and k-scaling

The condition admits the family $\theta_M = 15k$, $k \in \mathbb{Z}^+$, each k corresponding to k complete copies of the fundamental structure. While higher-k modes are valid overtones, $k = 1$ is the unique minimal mode and is taken as the physical base. The sign of k is fixed independently of minimality: by Axiom 2 the accumulated coordinate $|z_t|^2$ increases monotonically with t, so θ_M, which grades that same accumulating quantity, must be positive. The mirror solution at $k = -1$ is equally minimal in magnitude but excluded on this basis, not by convention.

5 Branch Physics

5.1 Scope and status

Section 4.2 describes the Planck scaffolding: the τ-loop, the base-B integer lock, and the spiral of primal boxes. That scaffolding is α-free. Every quantity entering it (M, T, P, V, L) is constructible without the fine-structure constant; only A and K carry α. This section concerns what happens *off* the scaffolding, in the domain where α does appear: the branch, or baryonic, domain.

This section is intended as the authoritative statement of branch mechanics. Material previously developed independently in the W-axis synthesis [8] and the quark/spin construction [10] is consolidated here; where those articles disagree with what follows, this section is to be taken as the correction. Results are labelled throughout as *derived* (following from the axioms and verifiable by direct computation), *consolidated* (established in [8] or [10] and re-verified here), or *postulated* (adopted structurally, not derived).

5.2 Two autonomous closure engines

The central structural claim of this section is that the branch is not a new mechanism. It is the same mechanism as the τ-loop, running with a different closure target.

The scaffolding τ-loop rotates and expands under a fixed rule set until it reaches its closure value, whereupon it collapses and resets. Its closure value is the render invariant

$$I = \pi^2\Omega^{15} = 341152.343709\ldots \tag{30}$$

The branch runs the same way, under the same kind of rule set, but does not close at I. For the electron it closes at

$$\psi = \frac{\sigma_e^3}{2T}, \tag{31}$$

where σ_e is the monopole amplitude defined in Section 5.3. Neither engine counts: each accumulates under its rule until a geometric condition is met. This is what is meant by calling both *autonomous*. The associated python script simulates a 'reduced' electron as ψ_e would require 10^{22} spiral steps (units of Planck time).

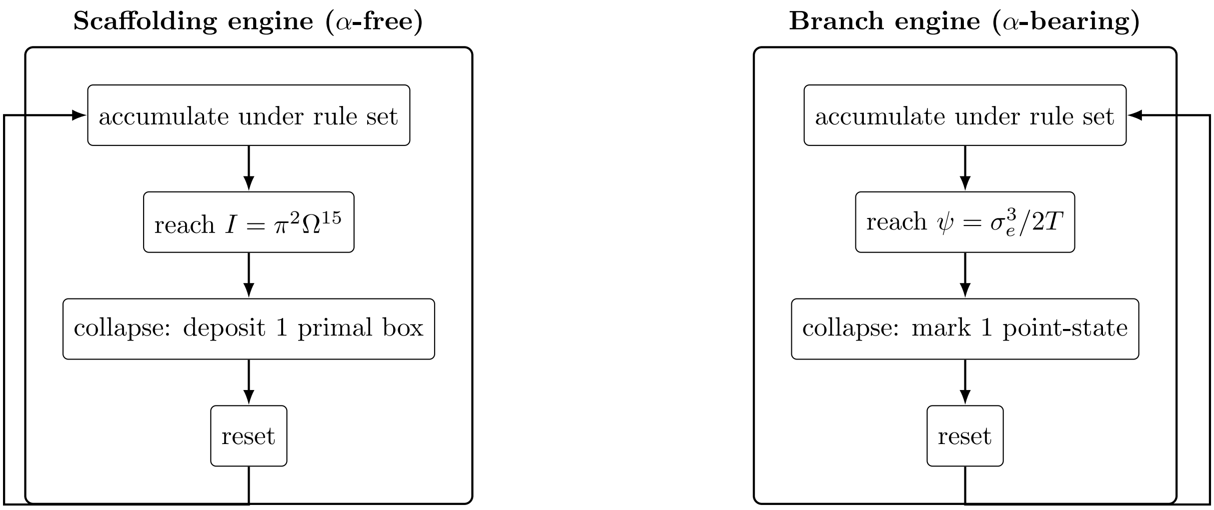

Figure 4: The two closure engines. Structurally identical; they differ only in closure target and in whether a new Planck unit is generated. The scaffolding engine deposits a primal box (Axiom 2); the branch engine does not generate new units and instead marks an existing point-state (Section 5.7).

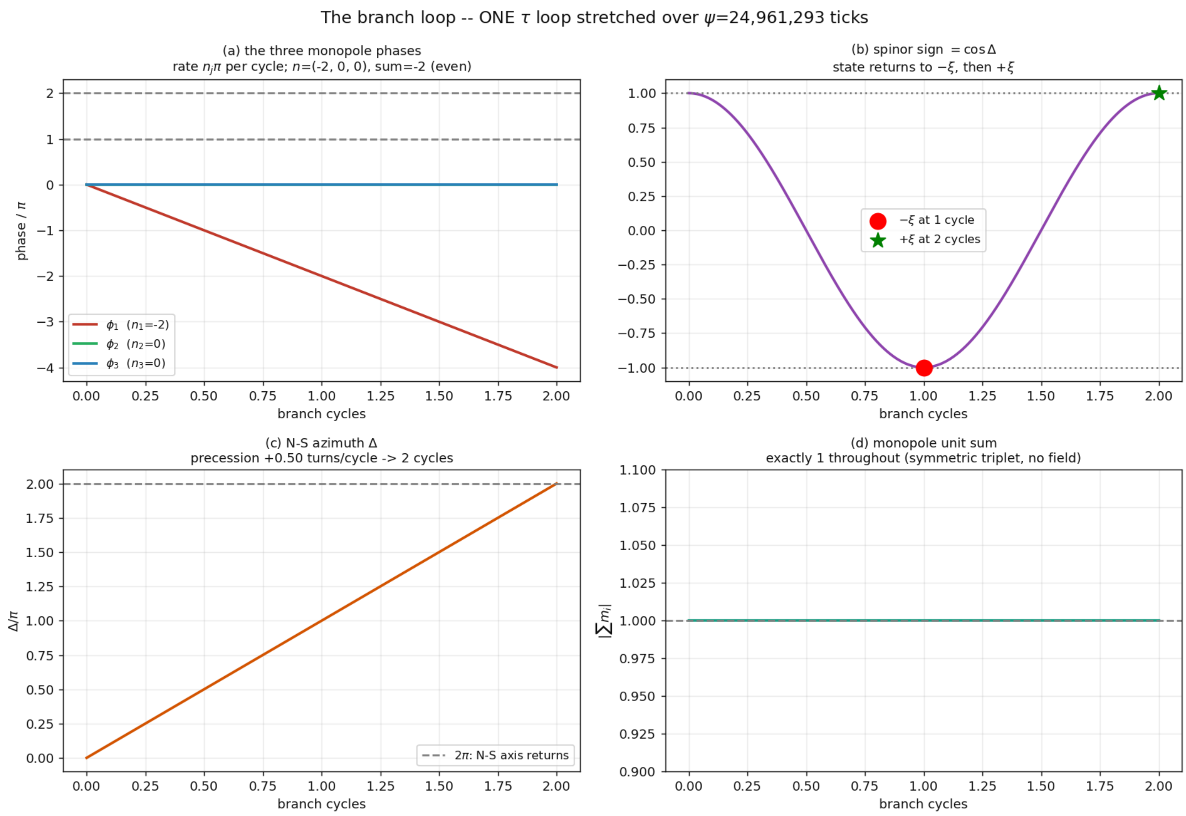

Figure 5: see spiral branch.py, *branch.png*

5.3 The monopole quantum Ω^5

Consolidated from [8], re-verified. The mass domain carries Ω^2 and the charge domain Ω^3; their tensor product is the dual-domain node

$$(\Omega^2)_{\text{mass}} \otimes (\Omega^3)_{\text{charge}} \;\cong\; \Omega^5, \qquad \Omega^5 = 32.574884\ldots \tag{32}$$

This is the smallest object with simultaneous presence in both domains, and is what [8] terms the Q^5 monopole.

Equation (32) has an immediate consequence for the scaffolding invariant. Since

$$\Omega^{15} = \left(\Omega^5\right)^3, \tag{33}$$

the Ω-content of the render invariant (30) is *exactly three monopole quanta.* The base-15 lock of Section 4.2 and the three-monopole electron of [10] are therefore not independent occurrences of the number three: they are the same factorisation seen from two sides. Section 3.3 records the further identity $I = \pi^2(\Omega^5)^3 = \mathrm{Vol}(\mathbb{RP}^3)$ at radius Ω^5.

The monopole amplitude. *Derived.* From the MLTA objects, with A the current ($\theta = 3$) and L the length ($\theta = -13$),

$$A\,L = 2^8\pi^5\alpha\,\Omega^5\,\frac{r^3}{v^2}, \qquad \theta(AL) = 3 + (-13) = -10, \tag{34}$$

which has the units of a magnetic monopole (ampere-metre). The normalised monopole amplitude is

$$\boxed{\sigma_e = \frac{3\alpha^{-2}AL}{2\pi^2} = 2^7\,3\,\pi^3\,\alpha^{-1}\,\Omega^5\,\frac{r^3}{v^2} = 5.314940\times10^7, \qquad \theta = -10.} \tag{35}$$

The reduction from $\alpha^{-2}AL$ to α^{-1} is exact, since A itself carries one power of α; this has been verified symbolically. Note that α enters σ_e exactly once. This is the sole role of α in the construction, and it is what distinguishes branch from scaffolding.

5.4 Why ψ is a cube

We can now answer the question of why ψ has the geometry it does (adapted from Article 7.). Three separate lines converge on the same cube.

(i) Composition. *Consolidated from [10].* The electron is the triplet $DDD = (AL)^3/T$ with $D = \sigma_e$. The cube is the particle's own composition: three monopoles.

(ii) Coincidence count. *Consolidated from [8], applied here.* If the three monopole phases are approximately independent, each with per-tick coincidence probability $p \approx 1/\sigma_e$, then the triple-coincidence probability is $p^3 \approx \sigma_e^{-3}$ and the expected waiting time to a simultaneous coincidence is

$$\mathbb{E}[W_3] \approx \sigma_e^3 = 1.501396 \times 10^{23}. \tag{36}$$

This is exactly the construction [8] uses to obtain $\mathbb{E}[W_2] \approx r_\alpha$ and $\mathbb{E}[W_3] \approx r_\alpha^2$, carried over to three constraints. The cube is therefore not only compositional; it is the dimension of the coincidence problem.

(iii) Unit-number closure. *Derived.* The cube is also forced dimensionally. Only the combination $(AL)^3/T$ has vanishing unit number and unit scalars:

$$\theta\big[(AL)^3/T\big] = 3(-10) - (-30) = 0, \qquad \left(\frac{r^3}{v^2}\right)^3 \frac{v^6}{r^9} = 1. \tag{37}$$

No power other than three closes. Both have been verified symbolically.

Dividing the coincidence time by the time unit $2T = 2\pi$ gives the invariant:

$$\boxed{\psi = \frac{\sigma_e^3}{2T} = 2^{20}\,3^3\,\pi^8\,\alpha^{-3}\,\Omega^{15} = 2.389545388 \times 10^{22}, \qquad \theta = 0.} \tag{38}$$

Reading (38) against (35): α enters three times, once per monopole; Ω^{15} is three monopole quanta; 3^3 is the triplet normalisation cubed. *Every factor in ψ is one of the three monopoles contributing its share.* That is the answer to why ψ has this geometry.

A second, independent route. *Verified symbolically.*

With $\sigma_t = 3\alpha^{-2}AV/(2\pi^2) = 2^6 3\pi^2\alpha^{-1}\Omega^5$ ($\theta = +20$, the U monopole), the alternative composition DUU gives

$$\psi = (2T)\,\sigma_t^2\,\sigma_e = 2^{20}3^3\pi^8\alpha^{-3}\Omega^{15}, \tag{39}$$

identical to (38), with unit numbers $(-30) + 2(20) + (-10) = 0$ and scalars again cancelling to 1. Two distinct quark structures yield the same invariant. This is a non-trivial consistency check, not a restatement, and it is the formal sense in which electron and positron share one geometry.

5.5 The holonomy constraint and the degrees of freedom

Consolidated from [10], Appendix A.. The three monopole phases ϕ_1, ϕ_2, ϕ_3 are not independent. They satisfy

$$\phi_1 + \phi_2 + \phi_3 \; = \; \sigma_e^3 \quad (\mathrm{mod}\ 2\pi), \tag{40}$$

and with $\sigma_e^3 = 2\pi\psi$ from (38),

$$\phi_1 + \phi_2 + \phi_3 = 2\pi\,\psi \quad (\mathrm{mod}\ 2\pi) = 2\pi \cdot \mathrm{frac}(\psi). \tag{41}$$

Closure is therefore phase alignment: the loop closes when the three internal phases sum to a whole number of turns. Nothing counts to 10^{22} (the electron frequency ψ_e); the count is emergent, which is what a geometry with no number storage requires.

5.5.1 The phase convention is fixed by the model, not chosen

Section D evaluates the wave-state at the loop boundary and obtains $\Psi(e) = \Omega^2 e^{i\pi} = -\Omega^2$. One complete τ loop therefore multiplies the state by -1: the phase advance is π per loop, not 2π. This is not a modelling choice available to us; it is a consequence already derived in this paper.

Writing $\phi_j(k) = \pi n_j k/\psi$ with $n_j \in \mathbb{Z}$ (integer turns, required because every phase must be real at the point-state), the holonomy at closure reads

$$\sum_j \phi_j(\psi) = \pi \sum_j n_j \equiv 0 \quad (\mathrm{mod}\ 2\pi) \qquad \Longrightarrow \qquad S \equiv \sum_j n_j \;\text{ is EVEN.} \tag{42}$$

5.5.2 Precession, read off the dynamics

The N–S azimuth is $\Delta = \phi_3 - \frac{1}{2}(\phi_1 + \phi_2)$, so per cycle

$$\boxed{\text{precession (turns/cycle)} = \frac{1}{2}\left[n_3 - \frac{n_1 + n_2}{2}\right] = \frac{3n_3 - S}{4} \equiv \frac{D}{4}.} \tag{43}$$

This is read directly off Δ rather than asserted. With S even, D has the parity of n_3, giving

$\lvert D \rvert$	precession $D/4$	cycles to return	reading
0	0	never (no precession)	spin-0
2	$\pm\frac{1}{2}$	2	fermion
4	± 1	1	boson
odd	$\pm\frac{1}{4}$, $\pm\frac{3}{4}$	4	no physical counterpart

$D = 0$ requires $S = 3n_3$, which with S even needs n_3 even, and is satisfied by thirteen triples within $|n_j| \leq 3$ — for instance $(0,0,0)$, $(-1,1,0)$, $(-2,2,0)$ and $(-2,-2,-2)$. A zero-precession configuration therefore exists, and the reference implementation's `classify()` returns "spin-0" for it.

5.5.3 The degree-of-freedom count

The ansatz of [10], Appendix A, is

$$z_1 = \sqrt{\tfrac{2}{3}}\, e^{i(\phi_1+\phi_2)/2}, \qquad z_2 = \sqrt{\tfrac{1}{3}}\, e^{i\phi_3}. \tag{44}$$

It is important to state plainly what this does and does not deliver.

- z_1 depends on ϕ_1, ϕ_2 *only through their sum.* Distinct pairs with equal sum give identical z_1; the split between ϕ_1 and ϕ_2 is invisible to the state.
- $|z_1|$ and $|z_2|$ are fixed by hand at $\sqrt{2/3}$, $\sqrt{1/3}$, so the polar angle of the Bloch vector is frozen at

$$n_z = |z_1|^2 - |z_2|^2 = \tfrac{1}{3}, \qquad \theta = \arccos \tfrac{1}{3} = 70.5288°, \tag{45}$$

  independently of all three phases.

The ansatz therefore sees two quantities, $(\phi_1 + \phi_2)$ and ϕ_3; the holonomy fixes their total; *one* free parameter remains. The reachable states form a latitude circle on the Bloch sphere, not the sphere.

degree of freedom	**fixed by**	**carries**
the sum (1 DOF)	holonomy (40)	timing: ψ, hence m_e, λ_e, ω_e
the residual (1 DOF)	free	azimuth on a fixed cone — *not* a full spinor

5.5.4 Where spin-$\frac{1}{2}$ actually comes from

The double-valuedness is not supplied by (44), and it does not need to be. The 3D domain of this model is the surface of the expanding hypersphere, i.e. S^3, and

$$S^3 \cong SU(2), \qquad SO(3) = S^3/\{\pm 1\} = \mathbb{RP}^3. \tag{46}$$

A 2π rotation *observed within the 3D domain* is a closed loop in $SO(3)$; its lift to S^3 runs from x to $-x$ and closes only after 4π. No half-angle need be assumed, no coupling between an internal τ phase and an external rotation need be postulated, and there is no separate Euclidean space in which to rotate: the observer's own rotation is already a path in $SO(3)$, whose double cover is the space itself.

The same factor of two is already present in the render invariant of Section 3.3:

$$I = \pi^2 \Omega^{15} = \mathrm{Vol}\big(\mathbb{RP}^3\big) = \mathrm{Vol}\big(SO(3)\big) \text{ at radius } \Omega^5, \qquad \mathrm{Vol}\big(S^3\big) = 2I. \tag{47}$$

The render lands on the $SO(3)$ volume; the state space has exactly twice it. Four apparently separate factors of two — the τ loop's π-per-cycle half-cycle, mass seeing ψ while spin sees 2ψ, the observable axis being 2π-periodic while the configuration is 4π-periodic, and this volume ratio — are one double cover seen from four directions.

Limit of the argument. Topology establishes that spin-$\frac{1}{2}$ states are *available* on S^3; it does not establish which objects carry them. A photon, having no point-state, forms no rigid triplet and hence no $SO(3)$ orientation, and is correctly left with helicity alone. But the selection rule that assigns half-integer spin to some closing branches and integer spin to others is not supplied by topology and remains open.

5.6 The branch cycle

Consolidated from [10], § "Point (mass) state versus wave (phase) state". The electron is not a persistent object but an oscillation between two states.

Definition 1 (Branch cycle)**.** *Let k index ticks within one branch cycle.*

- ***Point-state**, $k = 0$, duration $1\,t_P$. The branch coincides with the spiral. Unit numbers and scalars cancel ($\theta = 0$); one Planck-mass unit is present. The electron has a position.*

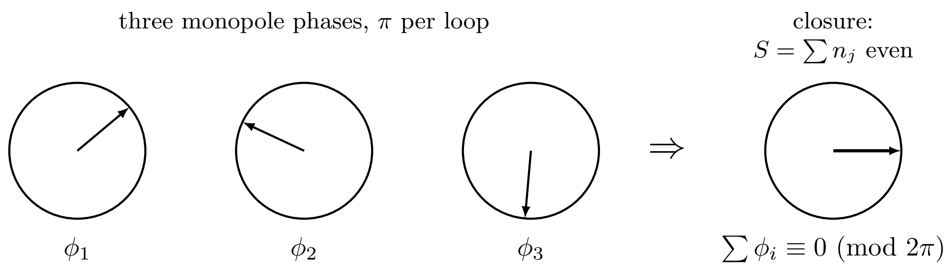

Figure 6: Branch closure as phase alignment. Each phase advances πn_j per cycle (Section D: one τ loop multiplies the state by -1), so the holonomy closes when $S = \sum n_j$ is even. The count ψ is the emergent number of ticks this takes, not a stored quantity. Verified in `branch_phases()` and `admissible()`.

- ***Wave-state**, $k = 1, \ldots, \psi$, duration $\psi\, t_P$. The branch is off the spiral, on the W-axis [8]. There is no mass density and no position. The degrees of freedom are the three monopole phases of* (40)*, evolving under $A(\tau) = \Omega^{\ln \tau + 1}$ (Axiom 8).*
- ***Closure**, at $k = \psi$:* (41) *is satisfied, the wave-state collapses, and the cycle restarts.*

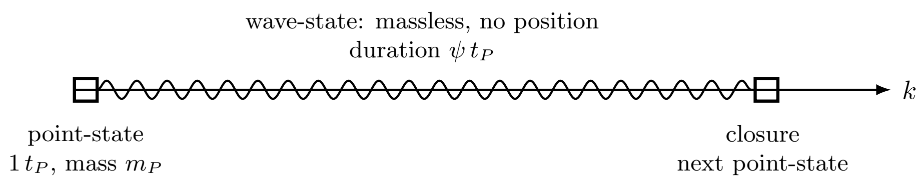

Figure 7: One branch cycle. The electron occupies a point-state for a single Planck tick and a wave-state for ψ ticks. The duty fraction $1/\psi = 4.18 \times 10^{-23}$ is what is observed as the electron's mass ratio.

Mass as a duty cycle. *Derived.* Because one Planck mass unit is present for one tick in every ψ, the time-averaged mass is

$$\langle m \rangle = m_P \cdot \frac{1}{\psi} = \frac{m_P}{\psi} = m_e. \tag{48}$$

The familiar relation $m = m_P/N$ is therefore not a scaling rule but a duty cycle: N is the number of ticks between successive point-states. This reading is required for consistency with Axiom 2, which admits only deposits of exactly one unit; a fractional deposit of m_P/ψ at collapse would violate it.

Observables. *Derived* All electron observables follow from ψ alone:

quantity	**expression**	**value**
mass	m_P/ψ	9.1082×10^{-31} kg
Compton wavelength	$2\pi\ell_P\psi$	2.4266×10^{-12} m
reduced Compton	$\ell_P\psi$	3.8621×10^{-13} m
angular frequency	$1/(\psi\, t_P)$	7.7624×10^{20} rad s^{-1}
frequency	$1/(2\pi\psi\, t_P)$	1.2354×10^{20} Hz
charge	AT	$\theta = -27$
duty fraction	$1/\psi$	4.1849×10^{-23}

5.7 Marking versus deposition

The branch generates no new Planck units. This distinguishes it sharply from the scaffolding engine, and requires care at the collapse event.

If the branch deposited a unit at closure, it would be generating scaffolding, which it does not. The consistent reading is that the point-state *marks* an existing primal box rather than creating one: at closure the branch coincides with a box the scaffolding engine has already deposited. Under this reading (48) is a duty cycle of marking, not of deposition, and Axiom 2 is untouched because branch closure is not a scaffolding collapse at all.

Two kinds of closure must therefore be distinguished:

	scaffolding closure	**branch closure**
target	$I = \pi^2\Omega^{15}$	$\psi = \sigma_e^3/2T$
α	absent	present, once per monopole
deposits box	yes	no
effect	spiral grows by 1	point-state marked

5.8 Autonomy of the branch clock

Derived; a constraint on admissible mechanisms. The electron mass is constant over cosmological time to high precision. Since $m_e = m_P/\psi$, the invariant ψ must be independent of the spiral index t.

This excludes a class of otherwise natural closure mechanisms. In particular, the branch phases cannot advance at the spiral's own phase rate $\arctan(1/\sqrt{t})$: were closure governed by that rate, the condition $\sum \omega\,\psi = 2\pi$ would place closure at $t \approx 1.3 \times 10^{44}$ and at no other epoch, so electrons could not form now. Equivalently, the number of spiral turns swept during one electron cycle falls from $\sim 5 \times 10^{10}$ at $t \sim 1$ to $\sim 10^{-40}$ at the present epoch, so any geometric intersection criterion tied to the spiral would have shut off long ago.

The branch clock is therefore autonomous: its rate is fixed by σ_e, an internal property, and the total phase rate is

$$\sum_i \omega_i = \frac{2\pi}{\psi} = \frac{4\pi^2}{\sigma_e^3} = 2.629448 \times 10^{-22} \text{ rad per tick.} \tag{49}$$

The three rates ω_i are constrained only by the holonomy, which fixes their sum, so they span a *two*-dimensional residual space. Exactly one combination of that space is visible in the state.

The distinction matters and has been stated inconsistently in earlier versions. The ansatz of Section 5.5 reads the phases only through $(\phi_1 + \phi_2)$ and ϕ_3; the combination $(\phi_1 - \phi_2)$ does not appear in it. Varying that combination at fixed sum leaves z_1, and therefore the entire state, numerically unchanged:

ϕ_1	ϕ_2	$\phi_1 - \phi_2$	z_1
0.600	0.600	0.000	$0.673884 + 0.461029\,i$
0.800	0.400	0.400	$0.673884 + 0.461029\,i$
1.100	0.100	1.000	$0.673884 + 0.461029\,i$

So the counting is:

three phases	3
holonomy constraint	−1
residual *dynamical* freedom (the ω_i)	**2**
of which visible in the state via the ansatz	**1**
invisible: $(\phi_1 - \phi_2)$	1

Describing the two residual ω_i as "the spinor" is therefore wrong, and conflicts with Section 5.5: a spinor requires two *state-visible* parameters covering the Bloch sphere, and the ansatz delivers one, confined to a latitude circle. The second dynamical degree of freedom is real but unobserved by the construction — which is a defect of the ansatz, not a hidden spinor.

5.9 Photon, positron, annihilation

Consolidated from [8], [10]. Within this framework the remaining cases are immediate:

- **Photon.** A branch whose τ-loop never closes. It has no point-state, deposits nothing, has no mass at any time, and propagates at $c = \ell_P/t_P$. It is the null case: the duty fraction is 0 rather than $1/\psi$.
- **Positron.** The DUU composition, $\theta = +30$, sharing the identical invariant (38) with DDD. Electron and positron have the same geometry and the same ψ, hence the same mass and frequency, differing in charge structure only.
- **Annihilation.** If e^- and e^+ interact such that the closure condition (41) can no longer be met, neither branch reaches a point-state again. Both become non-closing loops, i.e. photons. Annihilation is on this reading the destruction of a closure condition rather than the destruction of matter.
- **Marked versus unmarked boxes.** If the point-state marks an existing box (Section 5.7), the two are geometrically identical. As the box is already inclusive of a unit of Planck mass, we may describe the unmarked box as the absence of electron. The electron is then a purely branch-state phenomena, the spiral as the Planck unit scaffolding of the universe [3].

6 Conclusion

The CMB correlations of Article 1b are presented here not as numerological coincidences but as the macro-scale asymptotic results of a constrained minimal-complexity algorithm operating at the Planck scale. By modelling the universe as a continuous τ-function evaluating a hardcoded Ω equation, the need for pre-initialised constants and an externally imposed clock is removed. The discrete step and the mass unit $M = 1$ are forced by the geometry; the time unit $T = \pi$ follows from the minimal oscillation law of Axiom 5; and Ω is fixed by the single further postulate of Axiom 7. With two empirical inputs — α to open the electromagnetic channel and the anchor $f(y)$ to fix the internal scale [4] — the framework offers a candidate mathematical foundation for the Simulation Hypothesis.

A The Hypersphere

The architecture has one spiral — the Planck scaffolding of Sections 3.1–3.7 — and branches, which are particles and photons (Section 5). This section describes the geometry of the surface the spiral generates.

A.1 The 3D surface is S^3, and this is load-bearing

The observable 3D universe is the surface of the expanding hypersphere, i.e. a 3-sphere S^3, not a Euclidean $\mathbb{R}^3$. This is not a decorative choice of embedding. As unit quaternions,

$$S^3 \cong SU(2), \qquad SO(3) = S^3/\{\pm 1\} = \mathbb{RP}^3, \tag{50}$$

so a 2π rotation *observed within the 3D domain* is a closed loop in $SO(3)$ whose lift to S^3 runs from x to $-x$, closing only after 4π. Spin-$\frac{1}{2}$ is therefore available as a property of the space the model already posits, with no half-angle assumed and no coupling between internal and external rotations postulated (Section 5). There is no separate Euclidean arena in which to rotate: an observer's own rotation is already a path in $SO(3)$, whose double cover is the surface itself.

The same structure appears in the render invariant. From Eqs. (16)–(17),

$$I = \pi^2 \Omega^{15} = \mathrm{Vol}\big(SO(3)\big) \text{ at radius } \Omega^5, \qquad \mathrm{Vol}\big(SU(2)\big) = 2I, \tag{51}$$

so the render lands on the rotation group's volume and the state space is its double cover.

A.2 Expansion without a dark-energy term

An observer on the surface receives light from earlier states of the lattice, and the increasing radius produces a redshift indistinguishable from cosmic expansion. Writing $R \propto \sqrt{t}$ and taking elapsed time $\propto \sqrt{t}$, the scale factor grows linearly in elapsed time, $a \propto \sqrt{t} \propto$ time, giving

$$H = \frac{\dot{a}}{a} = \frac{1}{\text{time}}. \tag{52}$$

No dark-energy term is introduced: the apparent expansion is a geometric consequence of a surface whose radius grows. This is developed in the companion paper on hypersphere relativity [5].

A.3 The holographic reading

The spiral is a curve in the complex plane and the orthogonal phase rotation supplies one complex dimension, giving a local 2-D phase sheet. The fundamental degrees of freedom at any instant are therefore 2-D, and the box count is precisely the area (Section 3.7), with each primal box carrying π nats of Bekenstein–Hawking entropy.

Particles supply the third dimension. An electron's internal cycle spans $\psi_e \approx 2.39 \times 10^{22}$ Planck times, so it is not confined to a single 2-D slice; it is a coherent excitation across a radial thickness of order $10^{22}\,\ell_P$. Each tick contributes a 2-D phase slice; the electron's cycle stitches $\sim 10^{22}$ slices into a coherent voxel; lighter particles, with longer periods, require correspondingly greater thickness. On this reading the 3-D appearance of space arises from the finite temporal extent of particle wave-functions rather than from an intrinsically 3-D substrate: a 2-D hologram at the Planck scale, a 3-D volume at the Compton scale [11].

A.4 The arrow of time

The spiral is append-only. A deposited box is never rewritten, and a collapsed branch does not reopen. This yields a thermodynamic arrow without appealing to frozen shells: the past has lower effective entropy because it is fixed, while the present is the sole locus of writeable degrees of freedom.

The argument is worth separating from any claim about the local step rule. An append-only structure generates a directional history on its own, whatever the symmetry properties of the rule doing the writing.

B Bridging the Cosmological and Particle Scales

The apparent tension between Article 1a's $1/t^2$ density scaling and the τ-loop's r^3 volume terms dissolves once the two length scales generated by the Theodorus recursion are distinguished.

B.1 Two length scales from the Theodorus recursion

The Theodorus recursion

$$z_{t+1} = z_t S_t, \qquad S_t = 1 + \frac{i}{\sqrt{t}}, \tag{53}$$

produces two distinct natural lengths at every step t:

1. **Local radial scale.** The instantaneous radial coordinate of the spiral is

$$r_{\text{loc}}(t) = |z_t| = \sqrt{t}. \tag{54}$$

2. **Global cumulative arc length.** The step displacement has unit modulus, since $z_{t+1} - z_t = z_t\, i/\sqrt{t}$ and $|z_t| = \sqrt{t}$, hence $|z_{t+1} - z_t| = 1$. The total arc length traced by the spiral after t steps is therefore

$$s(t) = \sum_{k=1}^{t-1} |z_{k+1} - z_k| = t - 1 \;\simeq\; t. \tag{55}$$

B.2 Global density from the cumulative arc length

The model already identifies physical 3-space with the surface of an expanding hypersphere S^3 (Section A). On a 3-sphere of radius R, a great circle has circumference

$$s = 2\pi R. \tag{56}$$

If we interpret the total arc length $s = t$ generated by the spiral as the length of a great circle of the 3-sphere, then the global S^3 radius is fixed as

$$R_{\text{cosmo}} = \frac{s}{2\pi} \;\simeq\; \frac{t}{2\pi}. \tag{57}$$

This is not an arbitrary "cumulative arc length drives radius" assumption; it is the statement that the total path length, when closed into a loop, defines the equator of the 3-dimensional universe.

The 3-volume of S^3 at radius R is $2\pi^2 R^3$. Hence

$$V_{\text{cosmo}} = 2\pi^2 \left(\frac{t}{2\pi}\right)^3 = \frac{t^3}{4\pi} \;\propto\; t^3. \tag{58}$$

Mass is deposited linearly with the number of steps, $m_{\text{universe}} \propto t$, so the global density is

$$\rho = \frac{m_{\text{universe}}}{V_{\text{cosmo}}} \;\propto\; \frac{t}{t^3} = \frac{1}{t^2}. \tag{59}$$

The t^2 factor in the denominator is the area of the bounding 2-sphere associated with a radius $R \propto t$.

B.3 Particle-domain volumes

The particle-domain invariants such as

$$I = \pi^2\Omega^{15} = \mathrm{Vol}\big(\mathbb{RP}^3\big) \quad \text{at radius } \Omega^5 \tag{60}$$

are fixed volumes at fixed radii. They do not scale with the cosmological step t at all.

When a local 3-volume is constructed at the Theodorus radius $r_{\text{loc}} = \sqrt{t}$, it scales as

$$V_3(r_{\text{loc}}) \propto r_{\text{loc}}^3 = t^{3/2}. \tag{61}$$

This is the correct local scaling, but it is *not* the volume that appears in the cosmological density. The cosmological volume uses the global radius $R_{\text{cosmo}} \propto t$, not the local radius $\sqrt{t}$.

Thus the apparent contradiction between $r^3 \sim t^{3/2}$ and $\rho \sim t^{-2}$ disappears:

- $r^3 \sim t^{3/2}$ is a local particle-domain volume at the spiral radius.
- t^{-2} is the global mass-to-volume ratio after embedding the spiral's total arc length as a closed boundary on S^3.

They are different dimensional projections of the same discrete structure.

B.4 Status

What is established: the $1/t^2$ density requirement of Article 1 and the r^3 components of the τ-loop are mathematically unified by the Theodorus spiral, through the two length scales $r_{\text{loc}} = \sqrt{t}$ and $s = t$. The r^3 terms represent local 3D volumes at the spiral radius, while the $1/t^2$ density arises from the global 3-sphere volume whose radius is set by the great-circle length $s = t$.

C The Electron as a Delayed-Collapse Resonance

Section A describes a single spiral — the scaffolding — on which branches ride. Here we show that the mathematical electron, derived in prior work from purely geometric considerations, is a branch whose τ-loop runs slowly: the dimensionless ψ, constructed from the cross-section σ_e and the Planck time unit T, is identically the number of spiral steps that elapse before the branch closes.

C.1 Three closure behaviours

Everything in the simulation runs the identical hardcoded architecture: the Ω equation, the step operator $S_t = 1 + i/\sqrt{t}$, and the base-B closure. What differs is closure behaviour.

(i) **The spiral** (scaffolding, not a branch). The τ-loop closes at $\Delta \ln \tau = 1$, boundary $\tau = e$. Every B cycles one primal box is deposited and the spiral step t advances. This is the pacemaker.

(ii) **Photon branch.** The τ-loop has no closure condition and never completes. No point-state occurs, no box is ever marked, and the branch carries only phase. It propagates at $c = \ell_P/t_P$ and, having no point-state, does not advance along the expansion axis at all (Section A).

(iii) **Electron branch.** The τ-loop is the *same* loop, spanning $\Delta \ln \tau = 1$ as always, but executed ψ times more slowly: one complete loop takes ψ spiral steps rather than one. During the interval the branch is massless and has no position; at closure it briefly acquires both.

C.2 One slow loop, not many fast ones

Because the branch executes a single loop and has not completed it, it has not yet produced its first Ω^B. It therefore *cannot* deposit a Planck unit: the branch generates no new units, and the statement that it does so is not an additional rule but a consequence of the loop being unfinished. And because the scaffolding completes one loop per step while the branch completes one per ψ steps, the ratio of their outputs is ψ — which is exactly the mass ratio. *The mass ratio is the slowdown ratio*, not a separate fact requiring its own justification.

C.3 The dimensionless wave-function ψ

From the geometric electron construction,

$$T = \pi \frac{r^9}{v^6}, \qquad \theta(T) = -30, \tag{62}$$

$$\sigma_e = \frac{3AL}{2\pi^2\alpha^2} = 2^7\, 3\, \pi^3 \alpha^{-1} \Omega^5 \frac{r^3}{v^2}, \qquad \theta(\sigma_e) = -10, \tag{63}$$

where A is the current and L the length, so AL has the units of a magnetic monopole. The wave-function is the dimensionless ratio — both the θ grading and the r, v scalars cancel:

$$\psi = \frac{\sigma_e^3}{2T} = \frac{\left(2^7 3\pi^3\alpha^{-1}\Omega^5\right)^3}{2\pi} \qquad \Longrightarrow \qquad \boxed{\psi = 2^{20}\, 3^3\, \pi^8\, \alpha^{-3}\, \Omega^{15} = 2.389545 \times 10^{22}} \tag{64}$$

so ψ is fixed by the hardcoded constants together with the single empirical input α.

C.3.1 Three published forms, one invariant

With $\sigma_t = 2^6 3\pi^2\alpha^{-1}\Omega^5$ (the U monopole),

$$\sigma_e = 2\pi\, \sigma_t \tag{65}$$

exactly. Consequently the three expressions appearing across this series are the same quantity:

$$\underbrace{\frac{\sigma_e^3}{2T}}_{DDD} = \underbrace{(2T)\, \sigma_t^2 \sigma_e}_{DUU} = 2.389545 \times 10^{22}, \tag{66}$$

That two distinct quark structures (DDD and DUU) yield the identical invariant — with unit numbers $3(-10) - (-30) = 0$ and $(-30) + 2(20) + (-10) = 0$, and r, v scalars cancelling in both — is a non-trivial consistency check, not a restatement.

C.3.2 ψ is a radius

The factor $2T = 2\pi$ in (64) is a circumference-to-radius conversion, so σ_e^3 plays the role of a circumference and ψ of its radius. This is corroborated by the Compton lengths, which run the conversion back the other way:

$$\underbrace{\ell_P\psi = 3.8621 \times 10^{-13}\,\text{m}}_{\text{reduced (radius-like)}}, \qquad \underbrace{2\pi\ell_P\psi = 2.4266 \times 10^{-12}\,\text{m}}_{\text{full (circumference-like)}}, \tag{67}$$

against a measured full Compton wavelength of 2.42631×10^{-12} m. The recurring 2π is one conversion applied twice in opposite directions, $\sigma_e^3 \to \psi \to \lambda_C$.

C.4 Mass as a duty cycle

The electron is not a persistent object but an oscillation between two states: a point-state lasting one spiral step, and a wave-state lasting ψ. Because one Planck-mass unit is present for one step in every ψ, the time-averaged mass is

$$\langle m \rangle = m_P \cdot \frac{1}{\psi} = \frac{m_P}{\psi} = m_e. \tag{68}$$

The relation $m = m_P/N$ is therefore a *duty cycle*, not a scaling rule: N is the number of steps between successive point-states.
Under this duty-cycle reading no fractional deposit occurs: the observed m_e is a time average. And since the branch generates no new units at all (Section C.2), the point-state *marks* an existing box rather than depositing one — so branch closure is not a scaffolding collapse and the axiom's rule does not apply to it.

The remaining electron observables follow from ψ alone:

quantity	expression	value
mass	m_P/ψ	9.1082×10^{-31} kg
Compton wavelength	$2\pi\ell_P\psi$	2.4266×10^{-12} m
reduced Compton	$\ell_P\psi$	3.8621×10^{-13} m
duty fraction	$1/\psi$	4.1849×10^{-23}

Note on [10]. That article gives $\nu_e = \psi/t_P$, which evaluates to 4.43×10^{65} and is not a frequency of any physical quantity. The correct expression is $1/(\psi t_P)$, and it is the *angular* frequency, matching $m_e c^2/\hbar$. As printed the expression is in error by a factor ψ^2.

C.5 The mass spectrum as a delay spectrum

If every massive particle corresponds to a stable delay N,

$$m = \frac{m_P}{N} \quad \Longleftrightarrow \quad N = \frac{m_P}{m}, \tag{69}$$

then the mass hierarchy is the hierarchy of slowdown ratios. The extremes of the spectrum are:

N	mass	case
1	m_P	closes every step; no wave-state remains
ψ_e	m_P/ψ_e	electron
∞	0	photon: never closes

The photon has $N = \infty$, not zero: it is the infinite-delay case. Zero delay is $N = 1$, a Planck-mass object closing every step — the opposite extreme.

The $N = 1$ end has a further consequence, developed in [5]: since $m_{\text{obs}} = m_P$ is the ceiling, a particle's maximum Lorentz factor is

$$\gamma_{\max} = \frac{m_P}{m_P/\psi} = \psi, \tag{70}$$

so the velocity ceiling is *particle-dependent* — 2.39×10^{22} for the electron against 1.30×10^{19} for the proton, a ratio of 1836. Standard relativity has no such ceiling. The prediction is safe but untested: the most energetic particle ever observed reaches $\gamma \sim 10^{11}$, some eight orders of magnitude below the proton ceiling. The distinctive signature is not the ceiling's height but its mass dependence — a heavier particle meeting a limit a lighter one passes.

C.6 Summary

The electron does not *have* a Compton period; it *is* one. It is a branch running a single τ-loop ψ times more slowly than the scaffolding, massless and positionless throughout, marking a point-state once per ψ steps. Its mass is the time average of that duty cycle, its Compton wavelength the circumference corresponding to ψ as radius, and its stability the stability of the slow loop.

Because ψ is fixed by the hardcoded constants plus α alone, the electron introduces no free parameters. The sprial program emulates a 'reduced' electron; see: `branch_phases()`, `precession()` and `unit_sum()` of `spiral_branch.py`; the closure mechanics are treated in Section 5.

D Euler's Identity and the τ-Loop Boundary

The transcendental constants enter the architecture through two channels: π as the *wavenumber* of the standing-wave boundary condition (Axiom 5, whose eigenvalue is $\lambda_1 = \pi^2$), and e as the natural boundary of the unit growth law (Axiom 4). Here we show that Euler's identity $e^{i\pi} = -1$ is not an external curiosity but the exact phase factor the τ-loop evaluates at its collapse boundary. It is the result that *fixes the phase convention for the whole model.* Every downstream convention — the half-cycle of Axiom 5, the holonomy constraint on the turn triple, the two-cycle return that supplies spin-$\frac{1}{2}$ — descends from the single evaluation (75) below.

D.1 The standing wave as a complex exponential

The phase-generating oscillation is (Axiom 5)

$$w(\tau) = \sin(\pi \ln \tau), \tag{71}$$

the imaginary part of $e^{i\pi \ln \tau} = \cos(\pi \ln \tau) + i \sin(\pi \ln \tau)$.
Using $e^{a \ln b} = b^a$ the phase factor is compactly

$$\tau^{i\pi} = \cos(\pi \ln \tau) + i \sin(\pi \ln \tau), \qquad w(\tau) = \mathrm{Im}\big(\tau^{i\pi}\big). \tag{72}$$

D.2 Euler's identity at the loop boundary

Axiom 4 fixes the boundary at $\tau = e$ by requiring $\Delta \ln \tau = 1$. Evaluating (72) there,

$$\tau^{i\pi}\Big|_{\tau=e} = e^{i\pi \ln e} = e^{i\pi}, \qquad \text{and by Euler's identity} \qquad \boxed{e^{i\pi} = -1}. \tag{73}$$

The standing-wave node at the boundary, $w(e) = \sin(\pi \ln e) = \sin \pi = 0$, is therefore the point at which the full complex phase factor equals -1: a half-cycle inversion, not a full return.

D.3 The full wave-state at collapse

With the amplitude $A(\tau) = \Omega^{\ln \tau + 1}$ (Axiom 8), the complete state is

$$\Psi(\tau) = A(\tau)\, \tau^{i\pi} = \Omega^{\ln \tau + 1}\, \tau^{i\pi}. \tag{74}$$

At the loop start the phase factor is unity and the state is real, $\Psi(1) = \Omega$. At the boundary,

$$\boxed{\Psi(e) = \Omega^2\, e^{i\pi} = -\Omega^2.} \tag{75}$$

The collapse is thus the moment at which the real amplitude bridge (Ω^2) and the imaginary phase bridge ($e^{i\pi} = -1$) intersect. The orthogonal character of the collapse — real coordinate from imaginary phase — is the geometric counterpart of the π rotation that carries the accumulated imaginary phase onto the real axis.

Clarification. Equation (75) describes the closure of one τ-cycle. The deposition of a primal box occurs at the B-th such closure, when the internal counter completes and the spiral step t advances (Axiom 6), not at every cycle.

D.4 τ resets; the accumulated phase does not

This distinction is easily lost in prose and is the reason the double-valuedness exists at all, so it is stated explicitly.

Axiom 4 resets τ to 1 at each cycle boundary, so every cycle re-runs the same sweep $\tau : 1 \to e$. The *accumulated phase*, however, carries across cycles. Were it also to reset, each closure would independently return $-\Omega^2$, the state would never recover $+\Omega^2$, and there would be no two-cycle return — hence no spinor. Carrying the phase gives instead

$$\Psi \xrightarrow{\text{1 loop}} -\Omega^2 \xrightarrow{\text{2 loops}} +\Omega^2 \xrightarrow{\text{3 loops}} -\Omega^2 \xrightarrow{\text{4 loops}} +\Omega^2, \quad \ldots \tag{76}$$

The implementation follows this: `branch_phases(k)` returns $\pi n_j k/\psi$, accumulating in k, while τ itself restarts each cycle.

D.5 The consequence chain

Everything the model says about phase follows from (75):

1. $\Psi(e) = -\Omega^2$: one loop multiplies the state by -1, so the phase advance is π per loop, not 2π.

2. Two loops are therefore required to return, a 4π periodicity in the state against a 2π periodicity in anything observable (the modulus $|\Psi|$ is blind to the sign).

3. The holonomy closes when $\sum_j \phi_j \equiv 0 \pmod{2\pi}$. With $\phi_j = \pi n_j$ this requires

$$S \equiv \sum_j n_j \quad \text{EVEN}, \tag{77}$$

   which is the constraint used in Section 5.

4. The same factor of two appears geometrically: the 3D surface is $S^3 \cong SU(2)$, double-covering $SO(3) = \mathbb{RP}^3$, and the render invariant satisfies $I = \mathrm{Vol}(SO(3))$ with $\mathrm{Vol}(SU(2)) = 2I$ (Section A).

The τ-loop's half cycle and the $SU(2)/SO(3)$ double cover are not two facts but one, approached analytically in this section and topologically in Section A. Together they are the origin of spin-$\frac{1}{2}$, obtained without assuming a half-angle and without postulating a coupling between internal and external rotations.

D.6 The number 1 as the unit of reality

Euler's identity is conventionally written $e^{i\pi} + 1 = 0$, where the real number 1 bridges the imaginary rotation to zero. The same 1 is the unit of every structural operation in the lattice:

- Axiom 4: the growth law spans exactly $\Delta \ln \tau = 1$;
- Axiom 2: the real coordinate increment is exactly $\mathrm{Re}(S_t) = 1$;
- Section 3.6: the asymptotic action per step approaches $a_t \to 1$;
- Collapse: the deposited mass is exactly $M = 1$ Planck-mass unit.

D.7 The phase–amplitude bridge

Euler's identity connects π and e through the *imaginary* channel, $e^{i\pi} = -1$, i.e. $\ln(-1) = i\pi$. The capacity functional (Axiom 7) connects the same constants through the *real* channel,

$$\Omega^2 = \pi^e e^{1-e} = e\left(\frac{\pi}{e}\right)^e. \tag{78}$$

Writing (74) as

$$\Psi(\tau) = \underbrace{\Omega^{\ln\tau+1}}_{\text{amplitude bridge}} \times \underbrace{\tau^{i\pi}}_{\text{phase bridge}}, \tag{79}$$

the two converge at $\tau = e$ to the collapse state (75). Note the asymmetry in their epistemic status: the phase bridge is an identity of analysis, true independently of this model, whereas the amplitude bridge rests on the Tier 2 postulate of Axiom 7, including its evaluation point. Their meeting at $\tau = e$ is therefore a consequence of that postulate, not an independent confirmation of it.

D.8 Summary

Euler's identity is built into the τ-loop boundary condition: $w(\tau) = \sin(\pi \ln \tau)$ is the imaginary part of $\tau^{i\pi}$, which at $\tau = e$ evaluates to -1. The state at closure is $-\Omega^2$, so one loop is a half cycle and two are required to return.

This is the model's phase convention, derived rather than adopted, and the constraint S even, the 4π periodicity, and spin-$\frac{1}{2}$ all follow from it. The number 1 — the unit of real coordinate accumulation — is the lattice counterpart of the unit that completes Euler's identity to zero.

E Structural Parallels with Quantum Field Theory

The discrete counting geometry exhibits structural parallels with the foundational assumptions of QFT. Where QFT *assumes* continuous fields, quantization, and gauge symmetries as postulates, the τ-loop architecture offers a discrete geometric *origin* for some of them. The parallels differ greatly in strength, and the purpose of this section is as much to grade them as to state them.

Grading used below. *Structural* — follows from the axioms and is verifiable by direct computation. *Topological* — follows from the geometry of the posited space, but establishes availability rather than selection. *Conjectural* — a correspondence noted, not derived.

E.1 The imaginary domain and field excitations

(Structural.) In QFT, fields are continuous and carry complex phases prior to measurement. Here the imaginary component $i/\sqrt{t}$ generates a phase-rotation domain intrinsically, and the τ-loop's sweep of $A(\tau) = \Omega^{\ln\tau+1}$ is the geometric counterpart of an oscillating field. QFT places fields in a pre-existing background; this model generates the phase alongside the coordinate in a single complex recursive step, so field and background are not separate objects.

E.2 Collapse at Ω^2 and the origin of quantization

(Structural, resting on a Tier 2 postulate.) QFT postulates that fields quantize into discrete particles on measurement without deriving the mechanism. Here the wave-state saturates at the Ω^2 boundary (Axiom 8) — itself a structural postulate, not a derivation from nothing — and the continuous phase loop is forced to collapse into a discrete point-state. Quantization is on this reading a saturation boundary of the counting engine rather than an imposed rule.

E.3 Virtual versus real particles

(Structural.) In QFT virtual particles run in internal loops off-shell; real ones emerge on-shell. The model gives this a literal mechanism: the wave-state accumulates phase without marking a point-state while the loop is open, and becomes “real” only at closure. The duty cycle makes it quantitative — a particle is on-shell for a fraction $1/\psi$ of its existence (Section C.4).

E.4 The gauge symmetry spectrum

The Standard Model is organised around $U(1) \times SU(2) \times SU(3)$. The lattice supplies some representation-space structure but no gauge *dynamics*, and the three cases are not equally supported.

E.4.1 Global $U(1)$ phase symmetry

(Structural.) The phase of the step operator $S_t = 1+i/\sqrt{t}$ carries an intrinsic rotation $\arg(S_t) = \arctan(1/\sqrt{t})$. Note that S_t is not unit-modulus ($|S_t| = \sqrt{(t+1)/t} \neq 1$, since it mixes growth with rotation), so it is specifically the phase, not the operator, that furnishes a $U(1)$ action.

What the lattice provides is a *global* $U(1)$: the increment is the same function of t everywhere. Electrodynamics requires a *local* $U(1)$, $\psi \to e^{i\theta(x)}\psi$, demanding a site-dependent phase and a gauge connection. Promoting the lattice phase to a local symmetry is an open step; the global precursor is an exact consequence of the recursive complex step.

E.4.2 Three-state charge structure and $SU(3)$

(Structural for confinement-like exclusion; not established for colour.) From Section 4.2 the current unit-number is $\theta(A) = x/5$, evaluating with the integer lock $x = 15$ to $\theta(A) = 3$.

This is a *scaling exponent* — the power of Ω required to express current dimensionally — and is not by itself evidence of three physical entities. A single recurring integer across two contexts is not a correspondence. What [10] supplies is an actual three-component object: the electron as DDD, three monopole units, each individually excluded by a scalar non-cancellation rule (D alone carries scalars r^3/v^2, not 1), with only specific triplets — DDD, DUU, UDD — reducing to unit scalars when divided by T. That is a genuine, checkable confinement-*like* rule.

It should not be presented as a colour precursor. The DDD construction uses three copies of the *same* label, and the exclusion rule is dimensional cancellation, not projection onto a group-theoretic singlet. The correspondence is therefore narrower: a parallel to *why a lepton is built from three units*, not to *why each unit carries one of three colours*.

E.4.3 Spin-$\frac{1}{2}$

(Topological.) The 3D domain is the surface of the hypersphere, S^3, and

$$S^3 \cong SU(2), \qquad SO(3) = S^3/\{\pm 1\}, \tag{80}$$

so a 2π rotation *observed within the 3D domain* is a closed loop in $SO(3)$ whose lift to S^3 runs $x \to -x$, closing only at 4π. The same factor appears analytically as the τ-loop’s half cycle, $\Psi(e) = -\Omega^2$ (Section D), and numerically as $\mathrm{Vol}(SU(2)) = 2\,\mathrm{Vol}(SO(3)) = 2I$ (Section A).
What this does and does not establish. It requires no half-angle, no coupling postulate between internal and external rotations, and no ansatz. But topology is indiscriminate: it shows spin-$\frac{1}{2}$ is *available* on S^3, not which objects carry it. A photon, having no point-state, forms no rigid triplet and hence no $SO(3)$ orientation, and is correctly left with helicity alone — but the rule assigning half-integer spin to some closing branches and integer spin to others is not supplied by topology and remains open. The claim has therefore moved from “derived, including its dynamics” to “available by construction of the space, with selection unresolved”: weaker in one respect, considerably more secure in another.

E.4.4 Summary: precursors, not dynamics

The lattice does not generate $U(1) \times SU(2) \times SU(3)$ gauge dynamics. In descending order of support: the global $U(1)$ phase is an exact structural consequence of the recursive step; spin-$\frac{1}{2}$ is available topologically from $S^3 = SU(2)$, with selection unresolved; the three-component structure supports a confinement-like triplet rule but not a colour correspondence. The step from global discrete precursors to a local continuous gauge theory with self-interacting bosons remains a major open problem.

E.5 Particle ontology: slow closure versus *Zitterbewegung*

(Conjectural.) Dirac's theory predicts a trembling motion at the Compton frequency. Here the electron is a branch running one τ-loop ψ times more slowly than the scaffolding (Section C.2), with $\psi \approx 2.39 \times 10^{22}$ the number of spiral steps per closure. Its mass is the time average of the resulting duty cycle rather than a "cost". What QFT describes dynamically as self-interaction, the framework describes computationally as a slow loop. Recorded as a correspondence, not a derivation.

E.6 The arrow of time

Fock space is formally time-symmetric; the arrow in QFT is imposed by boundary conditions. The model offers a discrete analogue, and the well-supported half should be separated from a weaker claim.

E.6.1 Boundary conditions, not dynamics

(Structural.)

(i) **The monotonic counter.** The spiral step t advances monotonically and never resets. This is the foliation of the simulation — a fact about how it runs, not about the local rule's symmetry.

(ii) **Append-only deposition.** A deposited primal box is never rewritten. Information cannot flow backward to alter it. This is the discrete analogue of the Past Hypothesis: the past has lower effective entropy because it is fixed, while the present is the sole locus of writeable degrees of freedom.

(iii) **Closure asymmetry.** A branch that has closed and marked a point-state does not un-mark it. The realised history is not reversible, whatever the step rule's symmetry.

An append-only structure generates a directional history on its own, regardless of the symmetry of the rule doing the writing. That is the load-bearing part of this analogue.

E.6.2 The electron as illustration

During its slow loop the branch accumulates phase — the sweep of $A(\tau)$, the standing wave $\sin(\pi \ln \tau)$, the approach to $\tau = e$. Asymmetry enters at closure: the branch marks a point-state, and that marking is not undone.

E.6.3 Geometric asymmetry and the expansion attractor

It is tempting to claim the local rule $S_t = 1 + i/\sqrt{t}$ is itself time-symmetric, with the arrow of time entering solely through the global append-only structure (i)–(iii). However, the local step operator encodes the arrow geometrically.

While S_t is algebraically invertible, its modulus is strictly asymmetric: $|S_t| = \sqrt{(t+1)/t} > 1$ for all t, while $|S_t^{-1}| = \sqrt{t/(t+1)} < 1$. The forward step is strictly expansive, and its inverse is strictly contractive. Genuine time-symmetric microphysics requires the same *kind* of operation to describe both directions (as a unitary operator and its adjoint do). Here, the forward and backward operations are qualitatively distinct.

This mathematical asymmetry is a load-bearing feature, not a defect. It identifies lattice expansion as the forward-time attractor and contraction as the backward-time attractor. Macroscopic time-reversal in this geometry does not merely run the clock backward; it requires inverting the expansion scalar, forcing the entire lattice to shrink. The kinematic arrow of time is therefore inextricably locked to cosmological expansion.

Whether this local geometric contraction constrains the internal phase evolution of individual branches — for instance, by dictating how anti-particles relate to backward-traversing phase — remains a fertile area for the model. However, the fundamental asymmetry is already secured at the level of the step operator: forward time is expansion, backward time is contraction, and the two are not geometrically interchangeable.

E.7 Summary

The parallels are of markedly different strength, and the honest ordering matters more than the count:

parallel	**grade**	**note**
quantization as saturation	structural	rests on Axiom 8
virtual/real distinction	structural	quantified by the duty cycle
global $U(1)$ phase	structural	local $U(1)$ not obtained
arrow of time	structural	from append-only deposition
spin-$\frac{1}{2}$	topological	available, not selected
triplet stability	structural	confinement-like; not colour
Zitterbewegung	conjectural	correspondence only

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