

6. Natural Planck units MTP and the Fine-Structure Constant

Physical constant anomalies suggest a mathematical universe

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Abstract

We present a geometrical model in which the dimensioned physical constants are not independent empirical inputs but coupled projections of a compact dimensionless structure. An integer unit-number lattice with a base-15 closure replaces the independent base units (kg, m, s, A, K), and three Planck-like objects,

$$M = 1, \quad T = \pi, \quad P = \Omega = \sqrt{\pi^e e^{1-e}},$$

generate analogues of c , h , e , m_e , λ_e , R_∞ , G , k_B and γ_e . The electron sector reduces to a single dimensionless invariant ψ .

We treat the model as a tool for metrology. Two constants that were exact before 2019, c and μ_0 , and the Rydberg constant, determined from hydrogen spectroscopy to parts in 10^{12} , fix the lattice, and Ω is the only swing constant. No parameter is fitted, and the exponents of every formula are forced by the cancellation of the two SI translation scalars: the candidate set has 51 members, or 5.7 bits of search. The lattice reproduces α , e , h , m_e , λ_e and γ_e to parts in 10^8 ; G and k_B fall outside this regime. Expressed as shifts in Ω , the remaining misfits lie on one line, so the comparison with CODATA reduces to two numbers, an offset in α and an offset in h .

From 1998 to 2014, while c and μ_0 were exact, the lattice values moved by less than 4×10^{-12} between CODATA adjustments, and the lattice α has moved by less than 3×10^{-10} up to 2022. Against this reference the offset in α has stayed at 20 to 25 ppb while its uncertainty fell 24-fold. The offset in h appeared between 2006 and 2014 and is nearly 15σ for the value fixed in 2019; without that redefinition the lattice would place h between 6.626 068 7 and $6.626 069 7 \times 10^{-34}$ Js. Since 2019 the anchors c , e and R_∞ give α^{-1} independently of Ω , 29 ppb below the recommended value, and the significance of the disagreement is the same for either set of anchors. A QED-free determination of α with an uncertainty below 1×10^{-8} would decide between the lattice and the recommended value.

1 Introduction

“The physical constants form the scaffolding around which the theories of physics are erected, and they define the fabric of our universe, but science has no idea why they take the special numerical values that they do, for these constants follow no discernible pattern. The desire to explain the constants has been one of the driving forces behind efforts to develop a complete unified description of nature, or ‘theory of everything’. Physicists have hoped that such a theory would show that each of the constants of nature could have only one logically possible value. It would reveal an underlying order to the seeming arbitrariness of nature.”

— Inconsistent Constants, *Scientific American*, 292, p. 56 (2005)

*based on the Mathematical Electron model (<https://simulationuniverse.org/>)

Novel aspects:

1. Builds on a base-15 unit-number geometry.
2. Replaces independent SI base-unit assumptions with unit-number relationships: $kg(15)$, $m(-13)$, $s(-30)$, and $A(3)$, (item 1).
3. Assigns three primary geometrical Planck-like objects: $M = 1$, $T = \pi$, and $P = \sqrt{\pi^e e^{1-e}}$.
4. Derives geometrical analogues G^* , h^* , c^* , e^* , $(y_e/g_e)^*$, m_e^* , μ_0^* , and k_B^* via a unit-number lattice, (items 2, 3).
5. Reduces the electron sector to a dimensionless mathematical invariant ψ , from which m_e , λ_e , R_∞ , and y_e/g_e are linked, (items 2, 3).
6. Uses the lattice as a parameter-free reference for metrology, anchored on c , μ_0 and R_∞ (or c , e and R_∞ since 2019), and follows its agreement with CODATA from 1998 to 2022.

The dimensioned constants $G, h, c, e, m_e, \gamma_e, k_B, \mu_0$ show apparent anomalies that suggest an underlying mathematical structure, expressed here through geometrical objects (Planck unit analogues) for mass, time and (sqrt) momentum, MTP [1]. The anomalies do not lie in the SI numbers themselves, which depend on unit conventions, so a numberless, geometrical approach is used, in which the physical attribute (of mass, space, time, ...) is embedded in the geometry itself. The claim is that after a specified two-scalar translation from the geometric MTP sector to SI, dimensionless combinations and reconstructed constants agree with CODATA more closely than a coincidence model would allow, and this paper assesses how far that is evidence of a mathematical universe.

The model uses two dimensionless constants, the physical fine-structure constant α [9] and the mathematical constant Ω , a function of π and Euler's number $e = 2.718281828459\dots$ (Article 1b):

$$\Omega = \sqrt{\pi^e e^{1-e}} = 2.0071349543\dots$$

From mass $M = 1$, time $T = \pi$, square-root momentum $P = \Omega$ and α (the only physical constant required), we construct length L , velocity V and current A . These geometrical $MTPLVA$ objects are proposed as natural Planck units*, independent of any system of units, and the standard Planck-unit conversions give geometrical analogues $G^*, h^*, c^*, e^*, (y_e/g_e)^*, m_e^*, k_B^*$ of the SI constants with no free parameter.

*"...ihre Bedeutung für alle Zeiten und für alle, auch außerirdische und außermenschliche Kulturen notwendig behalten und welche daher als »natürliche Maßeinheiten« bezeichnet werden können..."
...These necessarily retain their meaning for all times and for all civilizations, even extraterrestrial and non-human ones, and can therefore be designated as "natural units"... -Max Planck [12]

The emphasis of latter sections is metrological. Section 4.3 fixes the lattice from c , μ_0 and R_∞ and compares its predictions with CODATA. Sections 5.2 to 5.3 express the misfit as a shift in Ω and follow it through the CODATA adjustments from 1998 to 2022. Section 6 treats the lattice as a reference and diagnostic for metrology, including the 2019 redefinition and the choice of anchors, and Section 7 shows that its formulas are forced and states the conditions under which they compress the constants.

2 Contextual Framework: From Arbitrary Constants to Geometric Order

Physical constants form the scaffolding on which the laws of physics are built, and they are conventionally treated as independent, arbitrary "givens" discovered by observation [11]. In the pre-2019 SI used by CODATA 2014, only c and μ_0 were exact, while h , e and k_B were adjusted measured constants; the 2019 redefinition fixed h , e , k_B and N_A as well, and μ_0 is now determined through α . Fixed values can be assigned only to constants that are independent of

one another, so the convention certifies the independence of their units, and in doing so it hides any mathematical dependency between them. In this geometric model only two SI anchors can be assigned independently, and the remaining constants are outputs of the lattice. CODATA 2014, the last adjustment in which only c and μ_0 were exact, is therefore the baseline of this analysis.

In this “Mathematical Universe” framework [2, 3], mass, space and time emerge from a unified, overdetermined mathematical structure. If the universe is a code-driven structure, the traditional independence of dimensions must give way to a fundamental relationship between units. This paper tests a geometrical unit-number approach in which the physical constants are outputs of a compact, generative constraint system rather than a collection of random parameters.

Note: To assign 4 constants exact values **is only possible if these constants are independent of each other**. It is argued here that **this premise needs to be questioned**.

3 Theoretical Foundation: The Unit-Number Mapping (θ) and Lattice Constraints

The core of the model is the unit-number relationship (θ), a mapping that breaks the presumed independence of SI units by assigning integer values to the base attributes [7]. Physical dimensions can then be treated as geometrical objects whose attributes are embedded in their geometry (the attribute of ‘length’, for example, is embedded in the geometry of the L object). The objects below are Planck unit analogues. Throughout this series the letter a denotes a geometrical analogue of the inverse fine-structure constant,

$$a \equiv \alpha^{-1} = 137.03599 \dots$$

3.1 The Base-15 Phase Closure

In Article 1b (tau-loop) the conversion from mathematical to geometrical (physical) units occurs through the symmetry

$$\frac{P^{15}T^2}{M^{12}} = \pi^2\Omega^{15}, \quad (1)$$

where each physical unit combines a geometrical object (the attribute) with dimensioned scalars, here r and v :

$$M = 1 \times \frac{r^4}{v}, \quad T = \pi \times \frac{r^9}{v^6}, \quad P = \Omega \times r^2 \quad (2)$$

$$\frac{P^{15}T^2}{M^{12}} = \frac{(\Omega r^2)^{15}(\pi r^9/v^6)^2}{(r^4/v)^{12}} = \pi^2\Omega^{15},$$

with r and v cancelling completely, so $\pi^2\Omega^{15}$ is the master scalar in the sense that matters here: it is independent of the internal (dimensioned) r, v gauge.

3.2 Scalars

Each object is assigned a dimensioned scalar, a numerical value with a unit; for example $M = (1)k = m_P$, $T = (\pi)t = t_P$, $V = (2\pi\Omega^2)v = c$. The unit-number rule θ applies to the scalars, so only two scalars need to be assigned values (numerical and units): the unit-number constraints fix all the others, and the system is exactly determined. The two chosen here are r ($\theta = 8$), which has only integer exponents, and v ($\theta = 17$), which follows exactly from $v = c/(2\pi\Omega^2)$. In SI units

$$v, \text{ units} = \frac{m}{s}, \quad r, \text{ units} = \left(\frac{kg \cdot m}{s} \right)^{1/4}$$

Attribute	Quantity	Object	Scalar	u^θ	unit
mass	M	(1)	$\frac{r^4}{v}$	u^{15}	kg
time	T	(π)	$\frac{r^9}{v^6}$	u^{-30}	s
sqrt momentum	P	$\Omega M^{(4/5)} (\frac{\pi}{T})^{(2/15)} = (\Omega)$	r^2	u^{16}	$\sqrt{\frac{\text{kg m}}{\text{s}}}$
velocity	V	$\frac{2\pi P^2}{M} = (2\pi\Omega^2)$	v	u^{17}	m/s
length	L	$VT = (2\pi^2\Omega^2)$	$\frac{r^9}{v^5}$	u^{-13}	m
current	A	$\frac{2^4 V^3 \alpha}{P^3} = (2^7 \pi^3 \alpha \Omega^3)$	$\frac{v^3}{r^6}$	u^3	A
temperature	K	$\frac{AV}{2\pi} = (2^7 \pi^3 \alpha \Omega^5)$	$\frac{v^4}{r^6}$	u^{20}	K

Table 1: P introduces Ω . The θ column is the unit-number invariant, and equals $8a + 17b$ for scalar content $r^a v^b$.

In Eq. (1) the scalars cancel, leaving a unitless geometrical form. Two further combinations in which both the scalars (v, r) and the unit numbers (θ) cancel are used in this model:

$$\frac{L^{15/2}}{M^{9/2} T^{11/2}} = \sqrt{2^{15} \pi^{19} \Omega^{30}}; \quad 15 * (-13) - 9 * 15 - 11 * (-30) = 0, \quad \frac{(\frac{r^4}{v})^9 (\frac{r^9}{v^6})^{11}}{(\frac{r^9}{v^5})^{15}} = 1$$

$$\frac{A^3 L^3}{T} = 2^{24} \pi^{14} \alpha^3 \Omega^{15}, \quad \text{units} = 1, \quad \text{scalars} = 1$$

In SI terms, with $L \equiv \text{m}$, $M \equiv \text{kg}$ and $T \equiv \text{s}$, the first combination identifies the unit group

$$f_x = \frac{kg^9 s^{11}}{m^{15}} \equiv \frac{M^9 T^{11}}{L^{15}}, \quad \text{units} = 1$$

with 1, and since $(r^9)^4 = kg^9 m^9 / s^9$, the scalars of L and T then carry the units of length and time:

$$\begin{aligned} \left(\frac{r^9}{v^5}\right)^4 &= \frac{kg^9 m^9}{s^9} \cdot \frac{s^{20}}{m^{20}} = \frac{kg^9 s^{11}}{m^{11}} = m^4 \frac{kg^9 s^{11}}{m^{15}} = m^4 f_x = m^4, \\ \left(\frac{r^9}{v^6}\right)^4 &= \frac{kg^9 m^9}{s^9} \cdot \frac{s^{24}}{m^{24}} = \frac{kg^9 s^{15}}{m^{15}} = s^4 \frac{kg^9 s^{11}}{m^{15}} = s^4 f_x = s^4. \end{aligned}$$

3.3 Diophantine System with Constraints

Initial conditions

$$\begin{cases} 15\theta_P + 2\theta_T - 12\theta_M = 0 \\ 2\theta_M + \theta_T = 0 \end{cases} \quad \theta_P, \theta_T, \theta_M \in \mathbb{Z} \setminus \{0\}.$$

Substituting $\theta_T = -2\theta_M$,

$$15\theta_P - 16\theta_M = 0.$$

Since $\text{gcd}(15, 16) = 1$, every integer solution is

$$\boxed{\theta_P = 16k, \quad \theta_M = 15k, \quad \theta_T = -30k, \quad k \in \mathbb{Z} \setminus \{0\}}. \quad (3)$$

Table 2: Physical constant analogues

Constant	Geometrical Object	Unit number θ from r, v
Speed of light	$c^* = V = 2\pi\Omega^2(v)$	17
Vacuum permeability	$\mu_0^* = \frac{4\pi V^2 M}{aLA^2} = \frac{a}{2^{11}\pi^5\Omega^4} (r^7)$	$8 \cdot 7 = 56$
Planck constant	$h^* = 2\pi MVL = 2^3\pi^4\Omega^4 \left(\frac{r^{13}}{v^5}\right)$	$8 \cdot 13 - 17 \cdot 5 = 19$
Gravitational constant	$G^* = \frac{V^2 L}{M} = 2^3\pi^4\Omega^6 \left(\frac{r^5}{v^2}\right)$	$8 \cdot 5 - 17 \cdot 2 = 6$
Elementary charge	$e^* = AT = \left(\frac{2^7\pi^4\Omega^3}{a}\right) \left(\frac{r^3}{v^3}\right)$	$8 \cdot 3 - 17 \cdot 3 = -27$
Boltzmann constant	$k_B^* = \frac{2\pi VM}{A} = \frac{a}{2^5\pi\Omega} \left(\frac{r^{10}}{v^3}\right)$	$8 \cdot 10 - 17 \cdot 3 = 29$

Pairwise distinctness holds automatically for every non-zero k . Since P carries r^2 and M carries r^4/v , $\theta_P = 2\theta_r$ and $\theta_M = 4\theta_r - \theta_v$, so $\theta_r = 8$ and $\theta_v = 17$, and

$$M^2T = \left(1\frac{r^4}{v}\right)^2 \times \left(\pi\frac{r^9}{v^6}\right) = \pi\frac{r^{17}}{v^8}, \quad \theta = 0. \quad (4)$$

The general solution and k -scaling. The condition admits the family $\theta_M = 15k$, $k \in \mathbb{Z}^+$, each k corresponding to k complete copies of the fundamental structure. While higher- k modes are valid overtones, $k = 1$ is the unique minimal mode and is taken as the physical base. The mirror solution at $k = -1$ is equally minimal in magnitude but excluded on this basis, not by convention.

3.4 Single anchor

Article 1c postulates a (embedded in the source code) primary constant $f(y)$ which numerically equals $1/M^2T = v^8/r^{17}$. This v^8/r^{17} scalar ratio has $\theta = 15 \cdot 2 - 30 = 0$, and so it has a numerical value but no units. If $f(y)$ is a universal constant, embedded rather than derived, then only one anchor can be specified: for defining v (from c) fixes r (from μ_0) through $f(y)$, and the same holds for any set of scalars. This restricts the number of independently assigned constants to 1, for once v is fixed then r is fixed and so μ_0 is fixed. Conversely if we choose μ_0 as our anchor constant then c is fixed by v . In other words, if we choose one constant as our anchor, we cannot assign exact values to any other constant.

$f(y)$ appears in units of Planck time when we calculate the CMB temperature at 1 K, and as $f(y)^2$ when the CMB reaches absolute zero. It is $f(y)$ that fixes the constraint $2\theta_M + \theta_T = 0$ as a structural postulate. Its value is not known independently and has to be estimated from the SI constants, which place it between 0.12300164 and $0.12300165 \times 10^{60}$ depending on whether r is taken from μ_0 , e or h , because the lattice does not fit all three (Section 6.4); with r from μ_0 and the fitted a of Section ?? it is $0.123001643 \times 10^{60}$. However, as we do not know the numerical value for $f(y)$ we have to use two anchors, which in CODATA 2014 are c and μ_0 .

By way of illustration, an alien civilization would not use SI units, but if we could decode its numerical value for the speed of light, we could calculate its velocity scalar v and from v we can calculate all its other constants, since its units are constrained by the same $f(y)$. This ‘universal language’ would give a means to communicate.

3.5 The Mathematical Electron: the ψ Invariant

The model defines the electron not as a physical particle but as a dimensionless mathematical invariant, ψ , constructed from magnetic monopoles (AL) and time (T). ψ is not itself a measured property of the electron; it is a pure number whose internal bookkeeping encodes the information needed to reproduce the physical electron. Because all units and scalars cancel in ψ , the electron observables (mass, charge, wavelength) appear as frequencies of the Planck unit objects.

$$\begin{aligned} T &= \pi \frac{r^9}{v^6}, \quad u^{-30} \\ \sigma_e &= \frac{3a^2 AL}{2\pi^2} = 2^7 3\pi^3 a \Omega^5 \frac{r^3}{v^2}, \quad u^{-10} \\ \psi &= \frac{\sigma_e^3}{2T} = 4\pi^2 \left(2^6 \cdot 3 \cdot \pi^2 \cdot a \cdot \Omega^5 \right)^3 \approx 0.23895452462 \times 10^{23} \\ \psi \text{ units} &= \frac{(u^{-10})^3}{u^{-30}} = 1, \quad \text{scalars} = \left(\frac{r^3}{v^2} \right)^3 \frac{v^6}{r^9} = 1 \end{aligned}$$

where σ_e has the units of a magnetic monopole (ampere-metre). The monopole AL has unit number -10 ($A = 3$, $L = -13$), so $(AL)^3$ gives -30 and cancels $T = -30$. The formula $4\pi^2(AL)^3$ has the form of the volume of a torus or the surface area of a 4-axis hypersphere, and divides into three magnetic monopoles, suggesting a ‘quark’ model of the electron with $D = AL$ constituents forming a DDD triplet. ψ adds four more constants, the electron mass, the Compton wavelength, the Rydberg constant R^* and the gyromagnetic ratio $(y_e/g_e)^*$:

$$\begin{aligned} m_e^* &= \frac{M}{\psi}, \quad \lambda_e^* = 2\pi L\psi, \\ R^* &= \frac{m_e^*}{4\pi L a^2 M} = \frac{1}{2^{23} 3^3 \pi^{11} a^5 \Omega^{17}} \frac{v^5}{r^9}, \quad u^{13} \\ (y_e/g_e)^* &= \frac{LA\psi}{4\pi MV} = \frac{2^5 \pi^3 \Omega^3 \psi}{a} \frac{1}{v^2 r}, \quad u^{-42}; \quad y_e = \gamma_e/(2\pi) \end{aligned}$$

3.6 Alpha, the Fine-Structure Constant

Algebraic consistency check (non-statistical): The standard relation $\alpha^{-1} = \frac{2h}{\mu_0 e^2 c}$, when the numerical constants are replaced by their geometrical analogues (h^* , μ_0^* , e^* , c^*), collapses exactly to return a . Both units and scalars cancel:

$$\begin{aligned} \alpha^{-1} &= \frac{2(h^*)}{(\mu_0^*)(e^*)^2 c} = \frac{2(2^3 \pi^4 \Omega^4)}{\left(\frac{a}{2^{11} \pi^5 \Omega^4} \right) \cdot \left(\frac{2^7 \pi^4 \Omega^3}{a} \right)^2 \cdot (2\pi \Omega^2)} = a \\ \text{units} &= \frac{u^{19}}{u^{56} (u^{-27})^2 u^{17}} = 1, \quad \text{scalars} = \frac{r^{13}}{v^5} \cdot \frac{1}{r^7} \cdot \frac{v^6}{r^6} \cdot \frac{1}{v} = 1 \end{aligned}$$

This is a deterministic pass/fail test of internal model consistency.

4 The unit number relationship

Through the MTP objects, the geometrical constants G^* , h^* , c^* , e^* , $(y_e/g_e)^*$, λ_e , m_e^* , k_B^* , μ_0^* can be expressed by π , α , Ω and two (SI-adjusted) scalars, v and r . Replacing v by c ,

$$v = \frac{c}{2\pi\Omega^2},$$

leaves a and r as the two floating variables, with c as the anchor. A fit determines the optimal aggregate values of a and r from the CODATA values of the six most precise constants, e^* , $(y_e/g_e)^*$, λ_e , m_e^* , R^* and μ_0^* , minimising the aggregate relative residual with each constant weighted equally within nested loops over a and r (Fig. 1); h contains no α and is left out lest it skew the result. Because μ_0 is one of the six fit inputs, its reproduction by the fitted (a, r) is a closure check on the fit and not an out-of-sample prediction; the parameter-free determination of μ_0 is the anchored one of Section 5.4, Eq. (7).

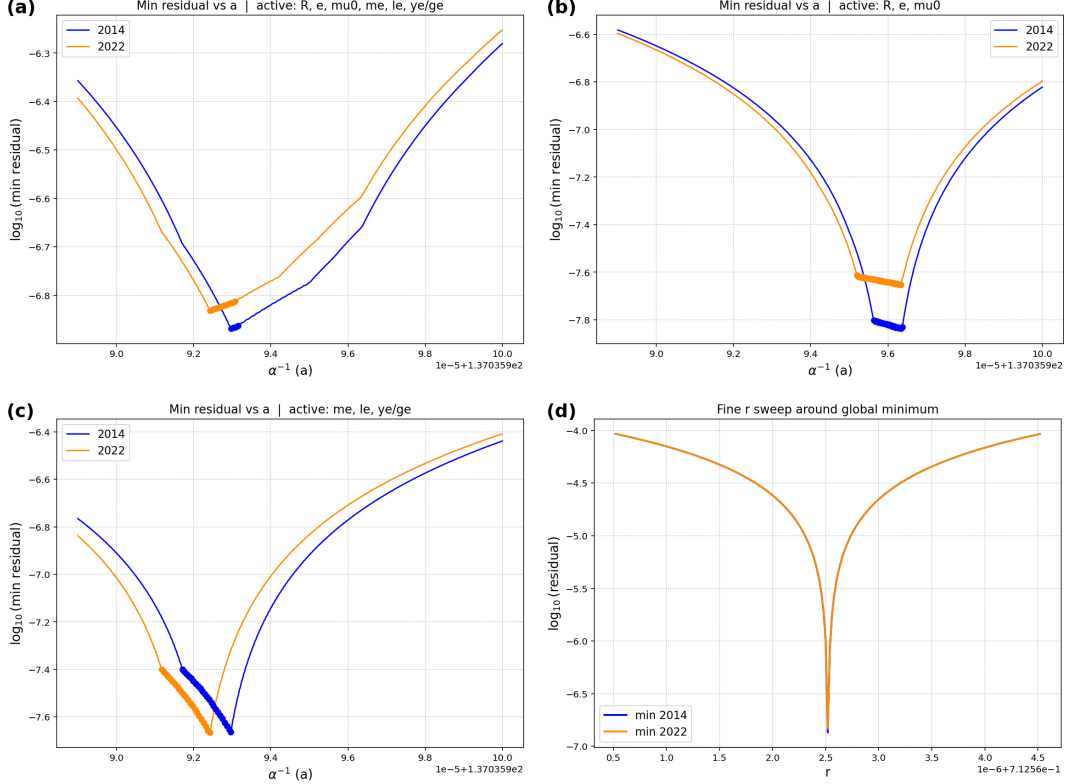


Figure 1: Alpha vs. residuals, CODATA 2014 (blue) and 2022 (orange). Minimum residual against a with (a) all six constants $e, R, \mu_0, m_e, y_e, \lambda_e$, (b) e, R, μ_0 and (c) m_e, y_e, λ_e ; (d) fine sweep in r around the global minimum of (a).

CODATA 2014 values

e	$1.6021766208(98) \times 10^{-19}$	m_e	$9.10938356(11) \times 10^{-31}$
h	$6.626070040(81) \times 10^{-34}$	μ_0	$4\pi/10^7$ (exact)
λ_e	$2.4263102367(11) \times 10^{-12}$	α^{-1}	$137.035999139(31)$
R	$10973731.568508(65)$	G	$6.67408(31) \times 10^{-11}$
$y_e = \gamma_e/2\pi$	$28024951640(170)$	k_B	$1.38064852(79) \times 10^{-23}$
g_e	$2.00231930436182(52)$		

The global best values of a and r are

constants	CODATA 2014		CODATA 2022	
	a	r	a	r
$e, R, \mu_0, m_e, y_e, \lambda_e$	137.035992962	0.71256252002	137.035992423	0.71256252091
e, R, μ_0	137.035995785	0.712562519	137.035995782	0.712562519
m_e, y_e, λ_e	137.035992962	0.71256252002	137.035992417	0.71256252092
h only	—	0.712562520005	—	0.712562520915

where the last row fixes r from $h^* = 2^3\pi^4\Omega^4r^{13}/v^5$ alone, with the CODATA 2014 mean and the 2022 assigned value of h . The constants divide into two groups, e, R, μ_0 and m_e, y_e, λ_e , and the six-constant result is that of the second group (identical in 2014, nearly so in 2022). The fit is skewed in favour of m_e, y_e, λ_e , so the constants cannot all be given equal weights.

4.1 Dimensionless Combinations

By reorganizing the constants into unit-free, scalar-free combinations, both the model unit numbers and the two SI translation scalars cancel. These combinations are not generally dimensionless in ordinary SI units; rather, they are dimensionless in the unit-number geometry, with $\theta = 0$. This distinction is important because in combinations where both units and scalars cancel, the SI constants should return the same numerical values as the *MTP* analogues. If the *MTP* objects are natural units, then in dimensionless combinations the SI constants will shed their scalar components, and reduce to the underlying embedded *MTP* objects (e.g.: $c = Vv = 299792458$ m/s and thus sans scalar v , $c = V = 2\pi\Omega^2$).

For example; h^3 , $\theta = 19 * 3 = 57$

$$(h^*)^3 = (2^3\pi^4\Omega^4\frac{r^{13}u^{19}}{v^5})^3 = \frac{2^9\pi^{12}\Omega^{12}r^{39}u^{57}}{v^{15}}, \theta = 57$$

$$\frac{2\pi^{10}(\mu_0^*)^3}{3^6(c^*)^5a^{13}(R^*)^2} = \frac{2^9\pi^{12}\Omega^{12}r^{39}u^{57}}{v^{15}}, \theta = 57$$

We then replace the geometrical object with the numerical SI (c, μ_0, R ; CODATA 2014 values of Section ??)

$$(h^*)^3 = \frac{2\pi^{10}\mu_0^3}{3^6c^5a^{13}R^2}$$

In this example we use h, e, c to derive the geometrical a which can then be compared with the CODATA α ;

$$\frac{(h^*)^3}{(e^*)^{13}(c^*)^{24}} = \left(\frac{2^3\pi^4\Omega^4u^{19}r^{13}}{v^5}\right)^3 / \left(\left(\frac{2^7\pi^4\Omega^3r^3u^{-27}}{\alpha v^3}\right)^{13} \cdot (2\pi\Omega^2vu^{17})^{24}\right) = \frac{\alpha^{13}}{2^{106}\pi^{64}(\Omega^{15})^5}$$

$$a = \left(\frac{(h^*)^3 2^{106}\pi^{64}\Omega^{75}}{(e^*)^{13}(c^*)^{24}}\right)^{1/13} = 137.03599594326, \alpha^{-1}/a = 1.0000000233$$

4.2 From c, μ_0, R, Ω .

Here we consider seven combinations of CODATA 2014 constants (**dimensionless according to the unit-number model; $\theta = 0$, dimensioned according to the SI system**). Except for R_7 , each combination comprises the 2 fixed constants c, μ_0 and 1 high-precision constant. They have SI units but $\theta = 0$ and scalars r and v have cancelled.

Note. we can also replace the geometrical a (the alpha component) with Ω by inserting this formula.

$$(a^*)^{26} = \frac{(c^*)^{35}}{2^{295}3^{21}\pi^{157}(\mu_0^*)^9(R^*)^7\Omega^{225}}$$

Table 3: Dimensionless combinations reduced to the geometric alpha a

	formula	a	$\Delta\alpha^{-1}/a$
(e)	$a = \left(\frac{2^{103}\pi^{64}\Omega^{75}\mu_0^3}{e^7 c^{21}} \right)^{1/10}$	137.0359949837	$1.0000000303 = 1+3.03\times 10^{-8}$
(λ_e)	$a = \left(\frac{\lambda_e^7 c^{35}}{2^{288}\pi^{157}3^{21}\Omega^{225}\mu_0^9} \right)^{1/12}$	137.0359931387	$1.0000000438 = 1+4.38\times 10^{-8}$
(R)	$a = \left(\frac{c^{35}}{2^{295}\pi^{157}3^{21}\Omega^{225}R^7\mu_0^9} \right)^{1/26}$	137.0359963688	$1.0000000202 = 1+2.02\times 10^{-8}$
(h)	$a = \left(\frac{2^{199}\pi^{128}\Omega^{150}\mu_0^{13}}{h^7 c^{35}} \right)^{1/13}$	137.0359927447	$1.0000000467 = 1+4.67\times 10^{-8}$
(γ_e)	$a = \left(\frac{(y_e/g_e)^7 \mu_0 c^{14}}{2^{178}\pi^{86}3^{21}\Omega^{150}} \right)^{1/15}$	137.0359915771	$1.0000000552 = 1+5.52\times 10^{-8}$
(m_e)	$a = \left(\frac{\mu_0^4}{2^{89}\pi^{29}3^{21}\Omega^{75}m_e^7 c^7} \right)^{1/25}$	137.0359929242	$1.0000000453 = 1+4.53\times 10^{-8}$

$$\begin{aligned}
 R_1 &= \frac{e^7 c^{21}}{\mu_0^3} & \text{SI: kg}^{-3} \text{m}^{18} \text{s}^{-8} \text{A}^{13}, \theta &= (-27) \cdot 7 + 17 \cdot 21 - 56 \cdot 3 = 0 \\
 R_2 &= \frac{\lambda_e^7 c^{35}}{\mu_0^9} & \text{SI: kg}^{-9} \text{m}^{33} \text{s}^{-17} \text{A}^{18}, \theta &= (-13) \cdot 7 + 17 \cdot 35 - 56 \cdot 9 = 0 \\
 R_3 &= \frac{R_\infty^7 \mu_0^9}{c^{35}} & \text{SI: kg}^9 \text{m}^{-33} \text{s}^{17} \text{A}^{-18}, \theta &= 13 \cdot 7 + 56 \cdot 9 - 17 \cdot 35 = 0 \\
 R_4 &= \frac{h^7 c^{35}}{\mu_0^{13}} & \text{SI: kg}^{-6} \text{m}^{36} \text{s}^{-16} \text{A}^{26}, \theta &= 19 \cdot 7 + 17 \cdot 35 - 56 \cdot 13 = 0 \\
 R_5 &= \left(\frac{y_e}{g_e} \right)^7 \mu_0 c^{14} & \text{SI: kg}^{-6} \text{m}^{15} \text{s}^{-9} \text{A}^5, \theta &= (-42) \cdot 7 + 56 \cdot 1 + 17 \cdot 14 = 0 \\
 R_6 &= \frac{m_e^7 c^7}{\mu_0^4} & \text{SI: kg}^3 \text{m}^3 \text{s}^1 \text{A}^8, \theta &= 15 \cdot 7 + 17 \cdot 7 - 56 \cdot 4 = 0 \\
 R_7 &= R^{21} \mu_0^{12} e^{35} & \text{SI: kg}^{12} \text{m}^{-9} \text{s}^{11} \text{A}^{11}, \theta &= 13 \cdot 21 + 56 \cdot 12 - 27 \cdot 35 = 0
 \end{aligned}$$

If the geometric model correctly describes nature then **there are no dimensioned components** (v and r have cancelled) to include in the calculations and so each (CODATA / Geometrical object) should closely approach unity at the model's residual scale (sans scalars the CODATA combination **is** the Geometrical object combination). This serves to validate the conjecture that the geometrical objects $M = 1, T = \pi, P = \Omega$ are natural Planck units [12], the above combinations would reduce to the same numerical values regardless of the system of units used, alien or terrestrial;

4.3 Note on Boltzmann Constant Variant

The Boltzmann constant k_B appears in two distinct measurement contexts in this analysis.

- $k_B = 1.380\,648\,52(79) \times 10^{-23}$ (CODATA 2014, thermal, from macroscopic thermometry)
- $k_B^* = 2\pi \cdot 2\pi\Omega^2 v \cdot 1r^4/v \cdot \frac{ar^6}{2^7\pi^3\Omega^3 v^3} = \frac{a}{2^5\pi\Omega} \left(\frac{r^{10}}{v^3}\right) = 1.379\,510\,194 \times 10^{-23}$
unit number = $17 + 15 - 3 = 29$

We may propose k_B^{**} , derived as an electromagnetic constant by using the electron gyromagnetic ratio $y_e = \gamma_e/2\pi = 28\,024\,951\,640(170)$ units = $\text{kg}^{-1}\text{s A}$ ($-15 + (-30) + 3 = -42$) and the g -factor $g_e = -2.00231930436182(52)$. This k_B^{**} closely matches the geometrical k_B^* :

$$k_B^{**} = \frac{g_e h}{4\pi c y_e m_e} = 1.379\,510\,310(24) \times 10^{-23}, \quad \theta = 19 - 17 - (-42) - 15 = 29$$

Rewriting the above formula gives

$$k_B^{**} = \left(g_e \cdot 2^3 \pi^4 \Omega^4 r^{13} / v^5\right) / \left(4\pi \cdot 2\pi\Omega^2 v \cdot y_e \cdot (1r^4/v)/\psi\right)$$

The thermal k_B discrepancy may indicate that the present electromagnetic-geometric mapping is not capturing the thermodynamic temperature sector, or that k_B^* corresponds to a different electromagnetic scale rather than the macroscopic thermodynamic Boltzmann constant.

Note: k_B^{**} is not the thermodynamic Boltzmann constant measured by macroscopic thermometry. It is a model-defined electromagnetic scalar constructed from electron-sector quantities and not an independent CODATA measurement of the thermal Boltzmann constant.

5 The Ω band analysis

The values of a in Table 3 spread over 35 ppb. With μ_0 exact in CODATA 2014, each combination compares the lattice with a single measured constant, so the spread is the misfit between the lattice and the measurements, expressed in a . This section expresses the same misfit as a shift in Ω , with the lattice anchored on c , μ_0 and R_∞ as a fixed reference. Section 5.1 sets out its predictions and their significance. Sections 5.2 and 5.3 show that the shifts form a band, too wide to be measurement scatter, that lies on a single line, and follow the line through the CODATA adjustments from 1998 to 2022. Section 5.4 rebuilds the lattice α from individual measurements. These results lead into the discussion of metrology in Section 6.

5.1 Predictions from c , μ_0 and R_∞

As in Section 4.2, the two unknowns are fixed exactly rather than floated as in Section ???. With c fixed, μ_0 and R_∞ determine a and r , and every other constant follows with no free parameter. In CODATA 2014 both c and μ_0 are exact, so the only measured input is R_∞ , known to 6×10^{-12} . Requiring the scalars r and v to cancel (Section 7.1) fixes the exponents, and we can replace the alpha component with this formula

$$a^{26} = \frac{c^{35}}{2^{295} 3^{21} \pi^{157} \Omega^{225} \mu_0^9 R_\infty^7}$$

giving the analogues of Table 3 in terms solely of c , μ_0 , R_∞ and Ω .

$$\begin{aligned} e^{13} &= \frac{2^{402} 3^{15} \pi^{231} \Omega^{300} R_\infty^5 \mu_0^{12}}{c^{64}}, & \lambda_e^{13} &= \frac{2^{282} 3^{21} \pi^{157} \Omega^{225} \mu_0^9}{c^{35} R_\infty^6}, \\ h^2 &= \frac{2^{99} 3^3 \pi^{59} \Omega^{75} \mu_0^5 R_\infty^5}{c^{15}}, & m_e^{26} &= \frac{2^{723} \pi^{453} \Omega^{525} \mu_0^{47} R_\infty^{25}}{3^3 c^{151}}. \end{aligned} \tag{5}$$

Two of these contain no a : e and h are predicted from c , μ_0 and R_∞ alone, so they test the lattice without reference to α . The other three pass through a , and they are not independent of one another, since $R_\infty \lambda_e = \alpha^2/2$ and $\lambda_e = h/m_e c$ hold both in the lattice and in CODATA. Table 4 compares the predictions with CODATA 2014 and 2022.

Table 4: Constants predicted from c , μ_0 and R_∞ (Eq. 5) with CODATA 2014 inputs, compared with CODATA. “Diff.” is (lattice/measured -1) in ppb; “dev.” expresses the 2014 difference in units of the 2014 uncertainty, combined with that propagated from R_∞ .

	lattice	CODATA 2014	diff. 2014	dev.	diff. 2022
α^{-1}	137.035 996 3688	137.035 999 139(31)	-20.2	-89σ	-20.4
e (10^{-19} C)	1.602 176 5977	1.602 176 6208(98)	-14.4	-2.4σ	-22.8
λ_e (10^{-12} m)	2.426 310 3347	2.426 310 2367(11)	$+40.4$	$+89\sigma$	$+40.9$
m_e (10^{-31} kg)	9.109 382 742	9.109 383 56(11)	-89.8	-7.4σ	-106.9
h (10^{-34} J s)	6.626 069 715	6.626 070 040(81)	-49.1	-4.0σ	-66.1

On the 2014 data the two predictions without a agree with measurement, e to 2.4σ and h to 4.0σ . The predictions that pass through a do not. The lattice a lies 20 ppb below the measured value, 89σ on the 2014 uncertainty; λ_e carries twice that offset through $R_\infty \lambda_e = \alpha^2/2$, and $m_e = h/\lambda_e c$ combines the offsets in h and λ_e . The five comparisons therefore contain two independent discrepancies, one in a and one in h , and the other three follow from them.

The 2022 column shows the effect of the 2019 redefinition. Between the 2014 adjustment and the 2017 special adjustment that fixed them, the measured h and e rose by 17 and 8 ppb, away from the lattice predictions, and since 2019 these values are exact. With their 2017 uncertainties (1.0×10^{-8} for h , 5.2×10^{-9} for e) the 2022 differences are 6.4σ and 4.4σ . With the 2019 values treated as exact they exceed 150σ , but that figure reflects the convention rather than new data (Section 6.3).

5.2 The Ω band

The predictions of Eq. (5) depend on three quantities besides c : μ_0 , R_∞ and Ω . A simple test asks whether a single correction to one of them could remove the misfit. Each is shifted in turn, $\mu_0 \rightarrow \mu_0 + \Delta\mu_0$, $R_\infty \rightarrow R_\infty + \Delta R_\infty$ or $\Omega \rightarrow \Omega + \Delta\Omega$, and the shift is solved for so that the prediction reproduces each measured constant. If one correction were responsible, the five shifts for that quantity would agree.

For μ_0 and R_∞ they cannot agree, whatever the data: the lattice satisfies $\alpha = \mu_0 c e^2/2h$ (Section 3.6) and $R_\infty \lambda_e = \alpha^2/2$ identically, as do the CODATA values, so a shifted μ_0 cannot reproduce α , e and h together unless $\Delta\mu_0 = 0$, and a shifted R_∞ cannot reproduce α and λ_e together. These two tests show that a discrepancy exists but cannot locate it.

Ω is the genuine test, since it is a constant of the model rather than a measured input. Replacing the alpha component by the geometrical a^* of Section 4.2 expresses each constant through c , μ_0 , R_∞ and Ω alone (Table 5). For a , e , h and m_e these are the relations of Eq. (5), so the ratios repeat the 2014 differences of Table 4, and the last two columns give the deviation from Ω required to derive each CODATA 2014 mean value.

Table 6 and Fig. 2 extend the shifts to CODATA 2018 and 2022, with the sign used in Table 4: a positive $\Delta\Omega$ means that the constant needs a larger Ω .

Four observations follow.

1. **No single correction to Ω removes the misfit.** With c , μ_0 and R_∞ fixed, a larger Ω raises both the lattice α (as $\Omega^{8.65}$) and h (as $\Omega^{37.5}$). The data require α lowered and h raised, so the implied shifts change sign across the constants: negative for α and λ_e , positive for e , h and m_e .

Table 5: Constants expressed in terms of Ω and CODATA 2014 c , μ_0 , R

Formula*	Codata/Calculated	$\Delta\Omega$	$\Delta\Omega/\Omega$
$(a^*)^{26} = \frac{(c^*)^{35}}{2^{295}3^{21}\pi^{157}(\mu_0^*)^9(R^*)^7\Omega^{225}}$	1.0000000202	$\Omega - 0.4689 \times 10^{-8}$	-2.34 ppb
$(e^*)^3 = \frac{4\pi^5}{3^3c^4(a^*)^8R}$	1.0000000144	$\Omega + 0.1256 \times 10^{-8}$	+0.63 ppb
$(h^*)^3 = \frac{2\pi^{10}\mu_0^3}{3^6c^5(a^*)^{13}R^2}$	1.0000000491	$\Omega + 0.2629 \times 10^{-8}$	+1.31 ppb
$(m_e^*)^3 = \frac{16\pi^{10}R\mu_0^3}{3^6c^8(a^*)^7}$	1.0000000898	$\Omega + 0.8923 \times 10^{-8}$	+4.45 ppb
$(y_e^*/g_e)^3 = \frac{3^3c^4}{2^8\pi^8a^*R^2\mu_0^3}$	0.9999999251	$\Omega - 5.2136 \times 10^{-8}$	-25.98 ppb

Table 6: Shift in Ω implied by each constant: $\Delta\Omega/\Omega$ in ppb, where $\Omega + \Delta\Omega$ in Eq. (5) reproduces the measured value, as in Table 4. Uncertainties are 1σ from the CODATA uncertainties, ignoring correlations.

constant	CODATA 2014	CODATA 2018	CODATA 2022
a	-2.34 ± 0.03	-2.31 ± 0.02	-2.36 ± 0.02
λ_e	-2.33 ± 0.03	-2.31 ± 0.02	-2.36 ± 0.02
e	$+0.63 \pm 0.27$	$+0.96 \pm 0.01$	$+0.99 \pm 0.01$
h	$+1.31 \pm 0.33$	$+1.72 \pm 0.01$	$+1.76 \pm 0.01$
m_e	$+4.45 \pm 0.60$	$+5.17 \pm 0.02$	$+5.30 \pm 0.02$
band width	6.8	7.5	7.7

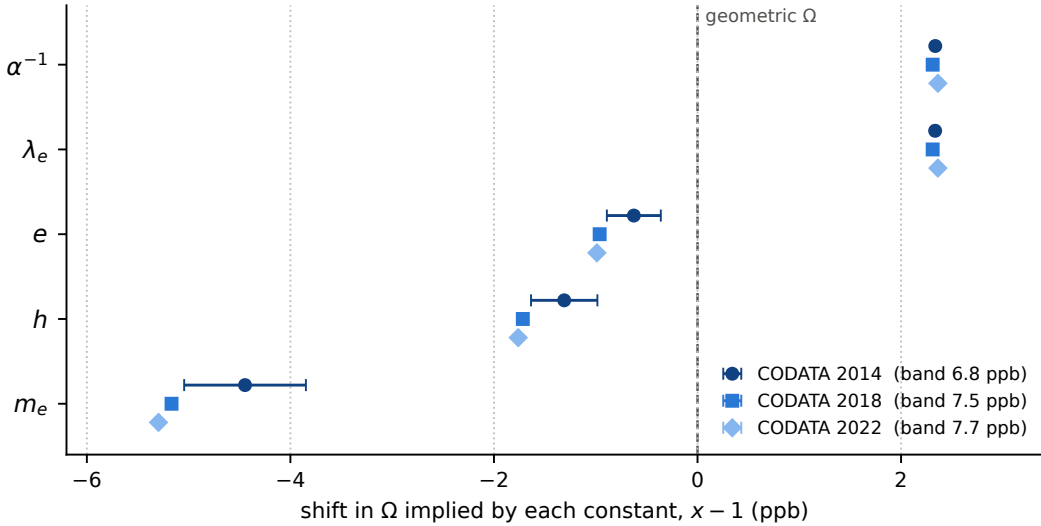


Figure 2: The Ω band. Each point is the shift $\Delta\Omega/\Omega$ that reproduces one CODATA constant from c , μ_0 and R_∞ (Table 6); the dashed line is the geometric $\Omega = \sqrt{\pi^e e^{1-e}}$. Error bars are 1σ . Since 2019 e and h are exact, so their 2018 and 2022 error bars reflect only μ_0 and R_∞ .

2. **The geometric Ω lies inside the band.** Each of the five CODATA 2014 constants is reproduced by an Ω within -2.3 to $+4.5$ ppb of $\sqrt{\pi e e^{1-e}}$. The shifts are small because Ω enters with large exponents (8.7 to 37.5, against 0.3 to 2.5 for μ_0 and R_∞), so the test is whether they agree, not how small they are.
3. **The band is not measurement scatter.** In 2014 its width, 6.8 ± 0.6 ppb, was eleven times its largest uncertainty.
4. **The band has not narrowed.** Its width went from 6.8 to 7.7 ppb between CODATA 2014 and 2022. The apparent thirty-fold fall in uncertainty over the same period comes mostly from fixing h , which cut the uncertainty of m_e from 12 to 0.3 ppb by definition. With the 2017 uncertainties restored to h and e , the 2022 band is 7.7 ± 0.5 ppb, no significant change from 2014.

Table 5 also includes γ_e , which the band leaves out. Since $y_e/g_e = e/4\pi m_e$, its lattice value varies only as $\Omega^{2.9}$, the exponents of e (23.1) and m_e (20.2) nearly cancelling, so the 75 ppb misfit it inherits from e and m_e needs a shift of -26.0 ± 2.1 ppb. With γ_e the band would run from -26 to $+4.5$ ppb, but its significance is unchanged: γ_e , e , h and m_e all lie 11σ from α (Section 5.3).

For these reasons CODATA 2014, the last full adjustment in which h and e were measured, is the baseline for the band. Since 2019 the band can move only through α : e and h are fixed, and μ_0 , λ_e and m_e all follow from α . It would close, to 0.6 ppb, if the measured α^{-1} were 137.035 994 8936, 31 ppb below CODATA 2022 and 150σ to 390σ from the individual Cs, Rb and electron $g-2$ determinations (Section 5.4). On the 2014 baseline two separate shifts would be needed: a by -20 ppb (89σ) and h by -49 ppb (4σ).

The band also depends on the anchors. With c , e and h fixed in place of c , μ_0 and R_∞ , the CODATA 2022 band runs from -1.9 to $+4.9$ ppb, and α implies $+4.9$ ppb rather than -2.4 . With c , R_∞ and e fixed, the lattice α (Pathway B) does not depend on Ω at all. The geometric Ω lies inside the band wherever the band is defined, but the Ω that a given constant implies is a property of the anchor set, not of the constant.

5.3 The Ω line

Table 7: Precision and lever of each constant, CODATA 2014, with c , μ_0 and R_∞ fixed. u_r is the CODATA relative standard uncertainty. With α and R_∞ fixed, the CODATA relations give $X \propto h^p$, and the lattice gives $X \propto \Omega^w$. $\sigma_\Omega = u_r/|w|$ is the precision the constant gives Ω , $\Delta\Omega/\Omega$ is the shift it implies (Table 6), and $D = (\Delta\Omega_X - \Delta\Omega_\alpha)/S$ is the offset of the CODATA h from the lattice h that it implies, with Ω set to reproduce α .

Constant	u_r (ppb)	p	w	$S = p/w$	σ_Ω (ppb)	$\Delta\Omega/\Omega$ (ppb)	D (ppb)
a	0.23	0	-8.65	0	0.026	-2.34	-
λ_e	0.45	0	17.31	0	0.026	-2.33	-
e	6.1	$\frac{1}{2}$	23.08	0.0217	0.27	+0.63	137 ± 12
h	12.2	1	37.50	0.0267	0.33	+1.31	137 ± 12
m_e	12.1	1	20.19	0.0495	0.60	+4.45	137 ± 12
γ_e	6.1	$-\frac{1}{2}$	2.88	-0.1733	2.10	-25.98	136 ± 12

Table 7 and Fig. 3 compare the shift in Ω implied by each constant with the precision of its CODATA value. The precision a constant gives Ω is its relative uncertainty divided by its exponent, $\sigma_\Omega = u_r/|w|$, so h , known to 12 ppb, fixes Ω to 0.33 ppb ($w = 37.5$), while γ_e , known to 6.1 ppb, fixes it only to 2.1 ppb, because $y_e/g_e = e/4\pi m_e$ varies as $\Omega^{2.9}$. e is twice as precise as m_e because it varies as $h^{1/2}$ rather than h , and the same power halves its share of the offset in h , so it lies closer to α .

The shifts lie on one line in the lever $S = p/w$,

$$\frac{\Delta\Omega}{\Omega} = -2.34 + 136.8 S \text{ ppb.} \quad (6)$$

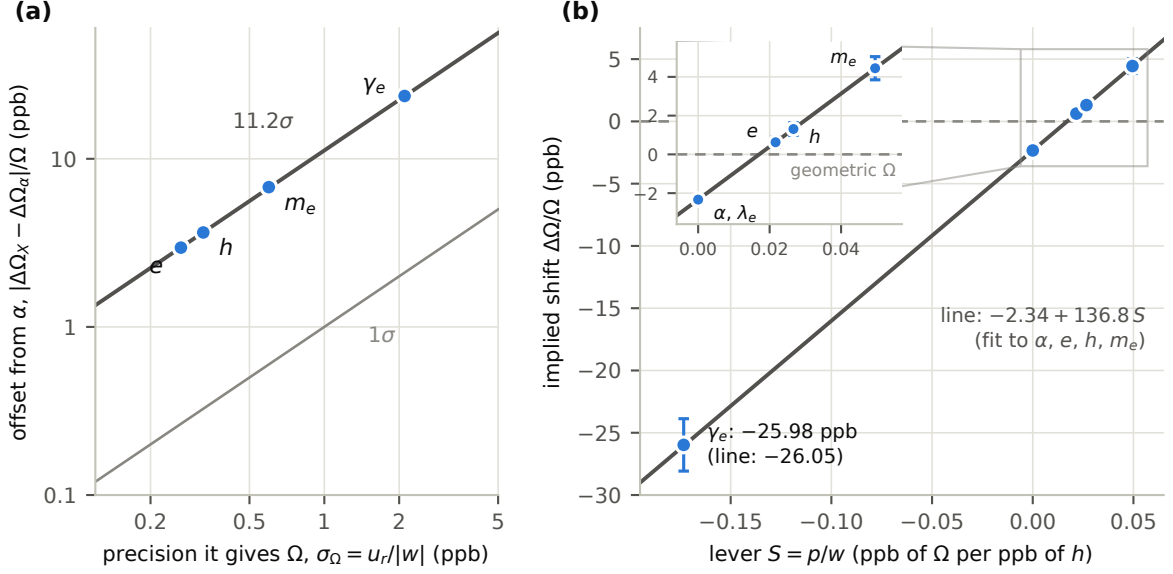


Figure 3: Shift in Ω against precision, CODATA 2014. (a) Offset of each constant's implied shift from that of α , against the precision the constant gives Ω . All four lie on the line at 11.2σ ; measurement scatter would put them near 1σ . (b) Implied shift against the lever $S = p/w$ (Table 7), with the line fitted to α , e , h and m_e ; γ_e is not used in the fit. Dashed: the geometric Ω . Error bars are 1σ , and the inset enlarges the region near $S = 0$.

The intercept is the shift that α requires. The slope, $D = 137 \pm 12$ ppb, is the amount by which the CODATA h exceeds the lattice h once Ω reproduces α , and every constant that carries h returns the same value of D (Table 7). Fitted to α , e , h and m_e , the line places γ_e at -26.05 ppb, and CODATA gives -25.98 ppb. γ_e is therefore not an outlier. Its shift is large because its exponent is small, and in units of its own precision it lies 11σ from α , as do e , h and m_e (Fig. 3a). Measurement scatter would put these four constants within a few σ of α , not at 11σ .

The line is straight because CODATA derives e , m_e and γ_e from h , α and R_∞ through relations that also hold in the lattice ($\alpha = \mu_0 c e^2 / 2h$, $R_\infty = \alpha^2 m_e c / 2h$, $\gamma_e / g_e = e / 2m_e$). Its straightness therefore tests those relations, not Ω . The test of the lattice lies in the two parameters of the line: the intercept, 89σ on the 2014 uncertainty of α , and the slope, 11σ . With CODATA 2022 the line is $-2.36 + 154.6S$ ppb. The intercept is unchanged, and the slope has grown by 18 ppb, mostly the 17 ppb rise in h fixed in 2019.

5.3.1 The line over CODATA history

If the band were measurement error, it would narrow as the measurements improve. Table 8 and Fig. 4 repeat the analysis for every CODATA adjustment since 1998 [31, 32, 33, 34, 14, 35, 16]. The lattice values depend only on c , μ_0 and R_∞ , and the lattice a moves by less than 3×10^{-10} between adjustments, against shifts of up to 5×10^{-9} in the measured α . The comparison therefore tracks how the measured values moved.

The two parameters of the line behave differently. The intercept, the shift that α requires, has lain between -2.36 and -2.28 ppb in every adjustment since 2002 except 2006 (-2.79 ppb), whose α carried an error in the QED theory of the electron magnetic-moment anomaly that was corrected in 2010 [34]. The 1998 value, -2.86 ± 0.42 ppb, agrees with the later ones within 1.4σ . In terms of a , the measured value has stayed 20 to 25 ppb above the lattice value (20 ppb since 2010) while the uncertainty of α fell 24-fold, from 3.6 ppb in 1998 to 0.15 ppb in 2022, so the significance of the offset has grown from 7σ to 126σ .

Table 8: The line of Eq. (6) for each CODATA adjustment since 1998. $\Delta\Omega_\alpha/\Omega$ is the shift in Ω that α implies with c , μ_0 and R_∞ fixed, and n_α is its size in units of its uncertainty. D is the CODATA h against the lattice h with Ω set to reproduce α , $D = \ln(h_{\text{CODATA}}/h_{\text{lattice}})$. For 2018 and 2022, where h and e are exact, the uncertainty of h from the 2017 special adjustment [15] is used.

CODATA	α^{-1}	h (10^{-34} J s)	$\Delta\Omega_\alpha/\Omega$ (ppb)	n_α	D (ppb)	D/σ_D
1998	137.035 999 76(50)	6.626 068 76(52)	-2.86 ± 0.42	7	-37 ± 80	-0.5
2002	137.035 999 11(46)	6.626 069 3(11)	-2.31 ± 0.39	6	$+24 \pm 167$	+0.1
2006	137.035 999 679(94)	6.626 068 96(33)	-2.79 ± 0.08	35	-9 ± 50	-0.2
2010	137.035 999 074(44)	6.626 069 57(29)	-2.28 ± 0.04	61	$+64 \pm 44$	+1.5
2014	137.035 999 139(31)	6.626 070 040(81)	-2.34 ± 0.03	89	$+137 \pm 12$	+11.1
2018	137.035 999 084(21)	6.626 070 15	-2.31 ± 0.02	123	$+151 \pm 10$	+14.5
2022	137.035 999 177(21)	6.626 070 15	-2.36 ± 0.02	126	$+155 \pm 10$	+14.8

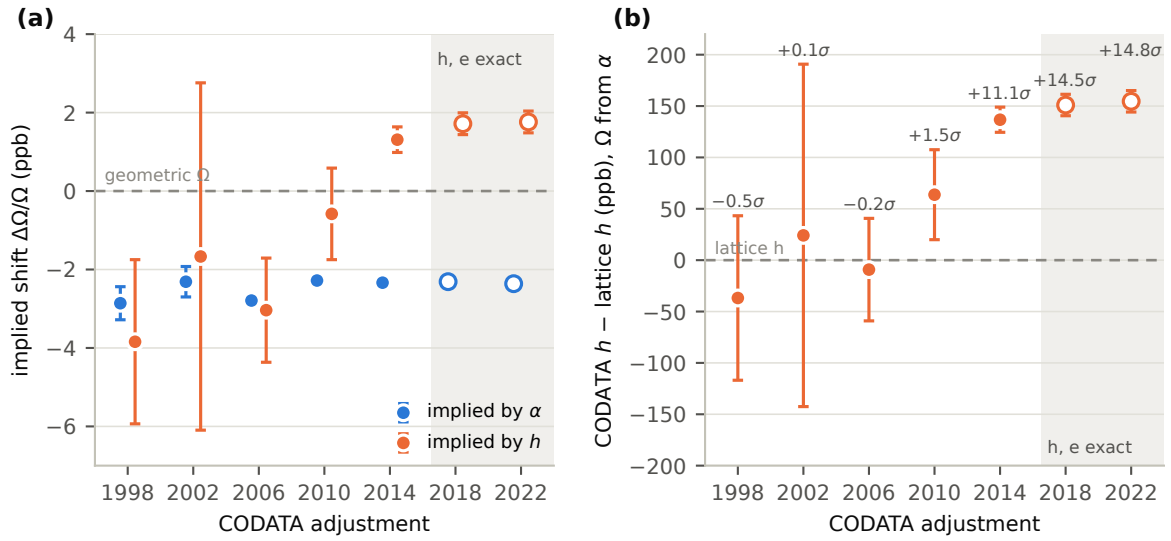


Figure 4: The line over the CODATA adjustments since 1998. (a) Shifts in Ω implied by α and by h , with c , μ_0 and R_∞ fixed; dashed, the geometric Ω . (b) The CODATA h against the lattice h , with Ω set to reproduce α (the slope D of Eq. (6)); labels give D/σ_D . Error bars are 1σ . Open symbols: 2018 and 2022, after h and e were fixed, with the 2017 uncertainty of h ; μ_0 then follows from α .

The slope tells a different story. With Ω set by α , the lattice h lies between $6.626\,069\,00$ and $6.626\,069\,15 \times 10^{-34}$ J s in every adjustment. It agreed with the CODATA h in 1998, 2002 and 2006 (within 0.5σ) and in 2010 (within 1.5σ). The CODATA value then rose: by 92 ppb in 2010, when a value of the Avogadro constant from enriched silicon replaced the earlier one from natural silicon [34], by a further 71 ppb by 2014, and by 17 ppb in the 2017 adjustment that fixed it [15]. The slope was 137 ± 12 ppb in 2014 (11σ), and it is 155 ± 10 ppb (14.8σ , with the 2017 uncertainty) for the value of h fixed in 2019. What this means for the CODATA record and the 2019 redefinition is taken up in Sections 6.2 and 6.3.

5.4 The value of α from raw-data baskets

Any three independent constants, or two together with c , fix a through their forced combination (Section 7.1); a pair with c is written $(X, Y | c)$. Table 9 lists the three pathways used most. Pathway A is blind to e and Pathway B to h , so if the lattice is correct the two must give the

same α . They agree in both years: on CODATA 2022 values they give 137.03599512277 and 137.03599520395, which differ by 8.12×10^{-8} , or 5.9×10^{-10} relative, 49 times less than either differs from CODATA. In raw-data terms they are not wholly independent, however: before 2019 e was derived from α and h , and since 2019 its value is the one fixed from them in 2017, so both pathways depend on the measured h with similar exponents. Pathway C sits apart because it combines μ_0 with e , and through $\alpha = \mu_0 c e^2 / 2h$ that combination carries part of the measured α .

Table 9: The three pathways: forced combinations of three constants (Table 13) that give α^{-1} with no fitted parameter, here with CODATA inputs.

pathway	combination	blind to	CODATA 2014	CODATA 2022
A	$\frac{c^{40}}{R_\infty^{26} h^{18}} = 2^{584} \pi^{254} \Omega^{450} 3^{78} \alpha^{-130}$	e	137.03599544	137.03599512
B	$c^4 R_\infty e^3 = \frac{4\pi^5}{27a^8}$	h	137.03599563	137.03599520
C	$R_\infty^{21} \mu_0^{12} e^{35}$ (R_7 , Section 4.1)	–	137.03599583	137.03599552

With CODATA values the pathways draw their inputs and their reference value from the same adjustment, and this is circular in a specific way. The lattice reproduces $\alpha = \mu_0 c e^2 / 2h$ identically (Section 3.6), so an input set that already satisfies that relation with CODATA’s α hands part of that α back to the calculation. A fairer test replaces each adjusted constant with a basket of individual determinations, and records how much each one relies on quantum electrodynamics (QED).

What is measured. Of R_∞ , e and μ_0 , only R_∞ is measured in its own right: since 2019, with h and e fixed at the values of the 2017 special adjustment [15], $\mu_0 = 2\alpha h / (e^2 c)$ is a measurement of α , and before 2019, with $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$ fixed, $e = \sqrt{2\alpha h / (\mu_0 c)}$ carried α and h . The baskets are therefore of R_∞ , h and α (Table 10), and e and μ_0 are rebuilt from them in either SI. Every R_∞ combines a hydrogen transition with the 1S–2S frequency through bound-state QED and a proton radius, so no QED-free R_∞ exists. Kibble balances measure h without QED, whereas silicon-sphere (XRCD) results give N_A and reach h only through α^2 . Among the determinations of α , the electron $g - 2$ depends on the QED series to five loops, and atom recoil uses QED only through R_∞ and the relative atomic mass of the electron, $A_r(e)$. The quantum Hall effect read against a calculable capacitor uses none, since $\alpha^{-1} = 2R_K / (\mu_0 c)$ with $R_K = h/e^2$.

Method. Each constant is a monomial in a and the scalars r and v , so its logarithm is linear in $\ln a$, $\ln r$ and $\ln v$. More than three constants are combined by weighted least squares, which gives an exact alternative to the grid search of Section ???. Uncertainties are propagated from the raw inputs, including the dependence of the lattice value on the measured α it is compared with. That dependence is weak. In the pre-2019 SI, Pathway B gives $\alpha \propto R_\infty^{1/8} (\alpha_{\text{exp}} h)^{3/16}$ and Pathway C gives $\alpha \propto R_\infty^{21/128} (\alpha_{\text{exp}} h)^{35/256}$, so the measured α enters with weights of only 0.19 and 0.14. The code and its full output are available [13].

Results. Figure 5 sets the lattice against the measurements. The electron $g - 2$ and both atom-recoil values lie within 1 ppb of CODATA 2022. Even so, Rb and Cs differ from each other by 1.2 ppb, or 5.5σ : two careful experiments disagree, and neither has been shown to be wrong. The QED-free values are two orders of magnitude less precise. NIST-97 lies 33 ± 24 ppb above CODATA, and the weighted mean of the five capacitor results 27 ± 18 ppb above. For the lattice, the R_∞ basket does not matter: its spread of 4×10^{-11} , which reflects the proton-radius puzzle, moves the lattice value by at most 0.01 ppb. The choice of α and h matters little: they enter

Table 10: The baskets, with standard uncertainties in the last digits.

Determination	Value	Method	QED content
<i>Rydberg constant</i> R_∞ (m^{-1})			
Beyer 2017 [17]	10 973 731.568 076(96)	H 2S–4P with 1S–2S	bound-state QED, r_p
Fleurbay 2018 [18]	10 973 731.568 53(14)	H 1S–3S with 1S–2S	bound-state QED, r_p
Grinin 2020 [19]	10 973 731.568 226(38)	H 1S–3S with 1S–2S	bound-state QED, r_p
Brandt 2022 [20]	10 973 731.568 332(52)	H 2S–8D with 1S–2S	bound-state QED, r_p
<i>Planck constant before 2019</i> , h (10^{-34} J s)			
NIST-4 2017 [21]	6.626 069 934(89)	Kibble balance	none
NRC 2017 [22]	6.626 070 133(60)	Kibble balance	none
IAC 2017, NMIJ	$N_A/10^{23}$ mol $^{-1}$:	silicon sphere (XRCD)	through α^2 and $A_r(e)$
2017, IAC 2015	6.022 140 526(70),		
[23, 24]	6.022 140 78(15), 6.022 140 70(12)		
<i>Inverse fine-structure constant</i> α^{-1}			
Fan 2023 [25]	137.035 999 166(15)	electron $g - 2$	heavy (to five loops)
Morel 2020 [26]	137.035 999 206(11)	Rb recoil, h/m	light (R_∞ , $A_r(e)$)
Parker 2018 [27]	137.035 999 046(27)	Cs recoil, h/m	light (R_∞ , $A_r(e)$)
NIST-97 [28]	137.036 003 7(33)	quantum Hall, calculable capacitor	none
NMI-97, NPL-88, LNE-01, NIM-95 [14]	R_K with relative uncertainties $4.3\text{--}13 \times 10^{-8}$	quantum Hall, calculable capacitor	none

only through e or μ_0 , with exponents no larger than $\frac{1}{2}$ in magnitude, and with any precise inputs the lattice stays between 19 and 37 ppb below CODATA. The pathways, however, differ from one another by up to 16 ppb, and this spread is not noise in the adjustment.

Where the spread comes from. With c fixed, the three pathways that use only R_∞ , e and h (A, B and $(e, h | c)$) agree to within 2 ppb, and the lattice then predicts

$$\mu_0 = 4\pi \times 10^{-7} (1 + 2.8 \times 10^{-8}) \text{ N A}^{-2}, \quad (7)$$

against the measured $4\pi \times 10^{-7} (1 - 1.3 \times 10^{-10})$ [16]. By $\alpha = \mu_0 c e^2 / 2h$ this is the discrepancy in α , stated for μ_0 . Pathways that use μ_0 inherit part of the measured α through it, and are shifted from the lattice value by an amount set by their exponents: towards the measurement for C and $(R_\infty, \mu_0 | c)$, away from it for $(e, \mu_0 | c)$. If μ_0 is replaced by the value implied by the lattice’s own α , the six pathways of Fig. 5 close up from a 16 ppb spread to 2 ppb. The lattice is therefore consistent with itself to a few ppb, and its offset from experiment reduces to a single number: the geometric μ_0 is 2.8×10^{-8} too large.

Significance. With raw inputs and propagated uncertainties (pre-2019 SI, Kibble-balance h), the discrepancy with the precise measurements is 5σ to 23σ for the five pathways that depend on h , directly or through e , because the 9–13 ppb uncertainty of the Kibble balances now enters. It is 99σ to 258σ for $(R_\infty, \mu_0 | c)$, which needs only R_∞ and the defined μ_0 . The 190σ quoted against the recommended uncertainty therefore overstates the case for some pathways and understates it for others. On every pathway the gap is 15 to 30 times the Rb–Cs disagreement, so it cannot be absorbed into the scatter between experiments, and it is not an artefact of the least-squares adjustment.

The QED-free comparison. With every input QED-free except R_∞ (Kibble h , and α from R_K), the lattice lies 2.2σ to 3.6σ below NIST-97 and 2.6σ to 4.3σ below the five-laboratory mean

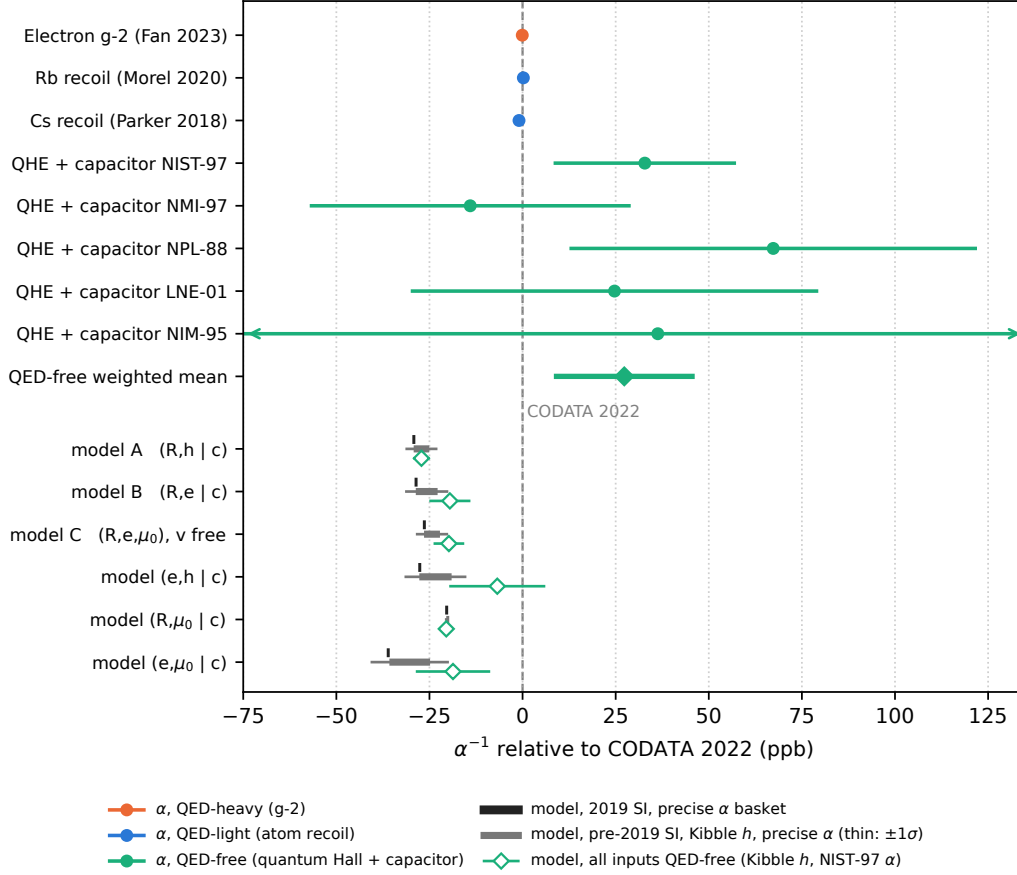


Figure 5: The lattice a from raw-data baskets compared with individual determinations of α , in ppb relative to CODATA 2022 [16] (dashed line). Top: the measurements, coloured by QED content (orange, electron $g - 2$; blue, atom recoil; green, quantum Hall effect against a calculable capacitor, which uses no QED), with 1σ error bars; the NIM-95 bar is clipped. Bottom: the lattice value for Pathways A–C and for three pairs $(X, Y | c)$, the pathway through X and Y with c fixed. Black: 2019 SI with the precise α basket (electron $g - 2$, Rb, Cs). Grey: pre-2019 SI with Kibble-balance h and the precise α basket (thick, span over the basket; thin, $\pm 1\sigma$). Open diamonds: every input QED-free except R_∞ .

(open diamonds in Fig. 5). The QED-free values lie on the far side of CODATA from the lattice, not on it. Atom recoil leaves little room for a QED offset either. There QED enters only through R_∞ and $A_r(e)$, and since $\alpha^2 \propto R_\infty/A_r(e)$, a 29 ppb shift in α would need a 58 ppb error in one of them, more than a thousand times its stated uncertainty. The bearing of these results on the reading of the lattice value as a ‘bare’ α is taken up in Section 8.2.

6 Discussion: Implications for CODATA and Metrology

Section 5.1 showed that two constants fixed by definition before 2019, c and μ_0 , one measured constant, R_∞ , and the geometric constant $\Omega = \sqrt{\pi e e^{1-e}}$ give the constants of the atomic and electromagnetic sector to parts in 10^8 , with no adjustable parameter, and Sections 5.2 and 5.3 expressed each remaining misfit as a shift in Ω . This section treats the resulting lattice as a tool for metrology: what it offers, what it shows about the CODATA record and the 2019 redefinition, how it can be anchored and tested, and where its reach ends.

6.1 The lattice as a metrology tool

Table 11 sets out what the lattice offers as a reference. Changes in the recommended values stand out against it: the two changes of 2010, in α and in h (Section 5.3.1), can both be read from Fig. 4. The stability itself is that of R_∞ and would be shared by any relation anchored on it. What the lattice adds is that it ties R_∞ to every other constant at once.

Table 11: The lattice as a reference for metrology.

Property	What the lattice provides	Section
Negligible input uncertainty	With c and μ_0 exact, only R_∞ is measured (5.9×10^{-12} in CODATA 2014; $a \propto R_\infty^{-7/26}$, $h \propto R_\infty^{1/2}$), so the lattice values are fixed to a few parts in 10^{12} , over a hundred times better than the measured α and a thousand times better than h .	5.1
Fixed reference	Moved by less than 4×10^{-12} from 1998 to 2014, while c and μ_0 were exact; the lattice α by less than 3×10^{-10} up to 2022; on c , e and R_∞ , by less than 2×10^{-13} from 2018 to 2022. The recommended α and h moved by up to 5×10^{-9} and 9×10^{-8} between adjustments.	5.3.1, 6.4
Common scale	Any measurement is a shift in Ω with precision $\sigma_\Omega = u_r/ w $: α (0.23 ppb) fixes Ω to 0.026 ppb, h (12 ppb) to 0.33 ppb, γ_e (6.1 ppb) only to 2.1 ppb, which ranks determinations by different methods, and future measurements, by how sharply they test the lattice.	5.3
Diagnosis by two numbers	The recommended values lie on one line, Eq. (6), with the shift that α requires as intercept and the offset of h as slope; a determination inconsistent with the others would lie off it. In CODATA 2014 the line, fitted without γ_e , places it within 0.07 ppb of its value.	5.3
QED bookkeeping	α follows along routes with heavy (electron $g - 2$), light (atom recoil, through R_∞ and $A_r(e)$) and no QED content (quantum Hall effect with a calculable capacitor), kept apart where an adjustment combines them.	5.4
Consistency by construction	$\alpha = \mu_0 c e^2 / 2h$, $R_\infty \lambda_e = \alpha^2 / 2$, $\lambda_e = h / m_e c$ and $\gamma_e / g_e = e / 2m_e$ hold identically, so a comparison with measurement tests only what the lattice adds to them.	3.6
Definite predictions	$h = 6.626\,069\,7 \times 10^{-34}$ J s in the pre-2019 SI; $\mu_0 = 4\pi \times 10^{-7} (1 + 2.8 \times 10^{-8})$ N A ⁻² (Eq. 7); $\alpha^{-1} = 137.035\,994\,894$, at which the Ω band would close in the 2019 SI. Each can be confirmed or excluded by measurement.	6.3, 5.4, 5.2
Transparency	Closed form from integer exponents, π and Ω , in a few lines of code [13], with no adjustment and no correlation matrix.	–

6.2 What the lattice shows about the CODATA record

CODATA determines the recommended values by a least-squares adjustment, which treats the constants as independent apart from established relations such as $R_\infty = \alpha^2 m_e c / 2h$. If the constants are tied by an overdetermined lattice, that independence is an assumption rather than a fact, and the adjustment has no way to represent the lattice relations. It does not appear to bias the recommended α , however: with raw inputs the lattice α^{-1} stays 19 to 37 ppb below the recommended value on every pathway (Section 5.4).

Against the fixed lattice, the record since 1998 separates into two parts (Section 5.3.1). The offset in α has been steady at 20 to 25 ppb while its uncertainty fell 24-fold, so its significance has grown from 7σ to 126σ . An offset that persists unchanged while the uncertainty falls this far is systematic: it belongs to the lattice, or to the relation between the lattice α and the

measured α , and not to the measurements. The offset in h is recent: the lattice h agreed with the recommended h in every adjustment up to 2010, and the offset appeared as the recommended value rose in 2010 and 2014. Neither offset has moved towards the lattice as the uncertainties have fallen, and the band has not narrowed (Section 5.2). The lattice cannot say by itself whether the measurements or the lattice are at fault; it says where they part, and when. The reading of the steady offset in α as a ‘bare’ value is taken up in Section 8.2.

6.3 The 2019 redefinition

Since 2019 the SI fixes h and e , together with k_B and N_A , at the values of the 2017 special adjustment [15]. This has two consequences for the lattice. First, a comparison with h and e is no longer a comparison with new data. Their 2017 uncertainties have moved into the realization of the kilogram and the ampere rather than disappeared, and no future measurement can move the fixed values towards or away from the lattice; the lattice h and e lie 66 and 23 ppb below the fixed values (49 and 14 ppb below the CODATA 2014 values, Table 4). Second, μ_0 is now measured, and since $\mu_0 = 2\alpha h/(e^2 c)$ it carries the measured α , so the lattice computed from c , μ_0 and R_∞ follows in part the α it is compared with, and the Ω band can move only through α (Section 5.2).

The value of h without the redefinition. In the pre-2019 SI, with c and μ_0 exact, the lattice predicts h from the two most precise measured constants, R_∞ and α . Because the lattice α differs from the measured one, the prediction depends on the anchors: 6.626 069 715 from μ_0 and R_∞ , 6.626 069 466 from μ_0 and α , 6.626 069 134 from all three with Ω adjusted by -2.34 ppb, and 6.626 068 747 from α and R_∞ , all $\times 10^{-34}$ J s. Without the redefinition the lattice would therefore place h between 6.626 068 7 and $6.626 069 7 \times 10^{-34}$ J s. Every CODATA value from 1998 to 2010 lies in this range, though with uncertainties of 44 to 166 ppb. The 2014 value lies 49 ppb above its upper end, and the 2017 value fixed in 2019 lies 66 ppb above it. The measurements behind the 2017 value [21, 22, 23, 24] all lie above the range: the nearest, the NIST-4 Kibble balance, by 33 ppb (2.5σ), and the NRC Kibble balance by 63 ppb (7σ). The redefinition did not create the offset in h . It fixed h at the measured value, and since 2019 the offset appears through α instead.

6.4 Choice of anchors

The lattice can be anchored on any constants that fix a and the scalar r once c fixes v . Before 2019 the natural choice is $\{c, \mu_0, R_\infty\}$: two exact constants and the most precise measured one. Since 2019 μ_0 carries the measured α , and the natural exact pair is c and e . Anchoring on c , R_∞ and e bridges the kinematic, atomic and electromagnetic domains, and e is preferred over h as the third anchor because e^* contains a whereas h^* does not, so that e gives a direct cross-check on α . In this combination, Pathway B of Table 9, the scalars r and v and the unit number θ all cancel,

$$R_\infty = \frac{c^5}{2^{28} 3^3 \pi^{16} \Omega^{27}} \frac{1}{a^5 r^9}, \quad e = \frac{2^{10} \pi^7 \Omega^9}{c^3} \frac{r^3}{a}, \quad c^4 R_\infty e^3 = \frac{4\pi^5}{3^3 a^8},$$

and rearranging isolates the fine-structure constant in a low-exponent formula that contains no Ω ,

$$\alpha^{-1} = \left(\frac{4\pi^5}{3^3 c^4 e^3 R_\infty} \right)^{1/8}.$$

With these anchors the lattice α is independent of the geometric constant and tests the structure of the lattice rather than the value of Ω , while Ω sets only h , μ_0 and m_e , each as Ω^{-25} .

With c and e exact since 2019, the formula gives α from R_∞ alone, to about 10^{-13} : $\alpha^{-1} = 137.035\,995\,20$ on CODATA 2022 values, 29 ppb below the recommended value. With the

geometric Ω the lattice then reproduces the fixed h to 4 ppb (1.2σ on its 2017 uncertainty), and it gives μ_0 2.5×10^{-8} above its measured value and m_e 62 ppb below its measured value, twice the offset in α since $m_e = 2R_\infty h/c\alpha^2$. With these anchors the whole disagreement sits in α .

Continuity from 1998. Before 2019 e was not a base constant. CODATA derived it from α and h , as $e = \sqrt{2\alpha h/\mu_0 c}$, and it moved by up to 49 ppb between adjustments and by 107 ppb from 1998 to 2019. A lattice built on each adjustment’s own e moves with it: its α by 3/8 of the change in e , 40 ppb over the period, and its h by 65/24 of it. It becomes a fixed reference in either of two ways: from 2019 on, when e is exact, or by applying the exact 2019 e to every adjustment. The present SI allows the second, because α and R_∞ do not depend on the definitions of the kilogram and the ampere, and it gives the lattice $\alpha^{-1} = 137.035\,995\,20$ in every adjustment. Its cost is that the exact e was set from the 2017 values of α and h , so the reference carries them, with weight 3/16 in α . For h the comparison would be circular, and Table 12 uses the exact e for α only.

Table 12: The offset in α with c , e and R_∞ as anchors: measured minus lattice α^{-1} , in ppb. With each adjustment’s own e the uncertainty combines those of α and h , which e carried before 2019; for 2018 and 2022 the 2017 uncertainty of h is used. With the exact 2019 e the lattice value is 137.035 995 20 in every adjustment.

CODATA	lattice α^{-1}	own e		2019 e
		offset (ppb)	offset/ σ	offset (ppb)
1998	137.036 000 72	-7.0 ± 15.0	-0.5	+33.3
2002	137.035 998 54	$+4.2 \pm 31.2$	$+0.1$	+28.5
2006	137.035 999 92	-1.7 ± 9.4	-0.2	+32.7
2010	137.035 997 42	$+12.1 \pm 8.2$	$+1.5$	+28.2
2014	137.035 995 63	$+25.6 \pm 2.3$	$+11.1$	+28.7
2018	137.035 995 20	$+28.3 \pm 2.0$	$+14.5$	+28.3
2022	137.035 995 20	$+29.0 \pm 2.0$	$+14.8$	+29.0

With the exact 2019 e the offset has stayed between 28 and 33 ppb since 1998, and at 28 to 29 ppb since 2010. With each adjustment’s own e , its significance repeats, adjustment by adjustment, that of the slope D in Table 8. Changing the anchors moves the disagreement from h to α , and changes the shift in Ω that each constant implies (Section 5.2), but not the size of the disagreement in units of σ , nor its history: absent up to 2006, and appearing between 2006 and 2014.

The two sets of anchors therefore serve different periods. $\{c, \mu_0, R_\infty\}$ is the natural base for the record before 2019, when c and μ_0 were exact: the lattice built on it moved by less than 4×10^{-12} from 1998 to 2014. $\{c, e, R_\infty\}$ is the natural base from 2019 on, with c and e exact and the electrical units realized from h and e through the quantum Hall and Josephson effects: the lattice built on it moved by less than 2×10^{-13} from 2018 to 2022. The change of base at 2019 follows the change the SI itself made, when it moved exactness from μ_0 to e . The numbers in this subsection are produced by `omega_band.py --eframe` [13].

6.5 Tests

Validating the lattice does not require metrological institutions to change their adjustment methods. It requires measurements that bear on the two offsets of Section 6.2, made without the dependences that an adjustment combines.

The offset in α . The decisive test is a QED-free determination of α . The quantum Hall effect read against a calculable capacitor gives $\alpha = \mu_0 c/2R_K$ with no perturbative input; since

2019 R_K is exact, so the test now runs through the capacitor, which realizes capacitance from $\varepsilon_0 = 1/\mu_0 c^2$. The existing determinations were made between 1988 and 2001 and lie on the far side of the recommended value from the lattice (Section 5.4). The most precise (NIST, 1997) has a relative uncertainty of 2.4×10^{-8} and lies 33 ppb above the recommended value, 2.6σ from the lattice value of Pathway B, and the mean of five laboratories gives 3.1σ [28, 14]. To separate the lattice and recommended values at three standard deviations would require an uncertainty below 1.34×10^{-6} in α^{-1} , that is 9.8×10^{-9} relative, roughly a factor of 2.4 beyond the best existing value. A result near the existing central values would exclude the bare/dressed reading of Section 8.2, and one near 137.0359952 would support it. Atom-recoil measurements of h/m , whose QED enters only through R_∞ and $A_r(e)$, already constrain the offset (Section 5.4).

The offset in h . Since 2019 h cannot be tested directly. The Kibble-balance and silicon-sphere determinations up to 2017 remain the evidence (Section 6.3), and in the 2019 SI the same offset is tested through α and μ_0 , by the closing value $\alpha^{-1} = 137.035\,994\,894$ and the prediction of Eq. (7).

The forced relations. The lattice reproduces $\lambda_e = h/m_e c$, $y_e/g_e = e/4\pi m_e$ and $R_\infty \lambda_e = \alpha^2/2$ as consequences rather than inputs, and Pathways A and B, blind to e and to h , as two routes to one α (Table 9). Had any of these failed, the model would already be refuted, and they remain a live constraint as measurements improve.

A value from a different direction. Article 4 [6] reaches a theoretical value of the inverse fine structure constant by a route that uses none of c , R_∞ , h , e or μ_0 : the requirement that an atomic transition end on an integer simulation step. For transitions to shells n not divisible by 3 to close on an integer step, the orbital period $T_1 = 8\pi\alpha^{-2}$ must lie on the grid $T_1 \in \frac{1}{3}\mathbb{Z}$, and the grid point nearest the measured α is $T_1 = 471964 + \frac{1}{3}$. We write the resulting value as

$$\alpha_{inv} = \sqrt{\frac{471964 + \frac{1}{3}}{8\pi}} = 137.03599583, \quad (8)$$

reserving α_{inv} for this theoretical value and α^{-1} for the standard constant as determined by each pathway of Table 9 or by CODATA. It lies 5.2, 4.6 and 2.3 ppb above Pathways A, B and C and 24.4 ppb below CODATA 2022.

Two qualifications bound what this shows. First, the measured α selects *which* grid point, although it plays no part in the value once the point is chosen. Adjacent grid points are 353 ppb apart in α^{-1} , so a value placed at random would lie as near to a grid point as the pathways do (4 ppb, taking their mean) only about 2% of the time: notable, but a single coincidence, to be weighed as one. Second, the grid admits only shells n not divisible by 3; requiring every observed level, $n = 3$ included, forces T_1 to be an integer and moves the value 348 ppb away. The agreement is therefore a consistency between independent routes, not a derivation of α .

6.6 Scope and limits

The tool works at the level of parts in 10^8 for the constants of the atomic and electromagnetic sector, which are intrinsic properties of single particles or of the vacuum. The constants measured on macroscopic or statistical systems fall outside it: combined with c , μ_0 and R_∞ in the same way as h^3 in Section 4.1, G , ℓ_P , m_P and k_B return values of a that differ from the recommended value by 4×10^{-5} to 5×10^{-4} . These deviations do not form a single group, and the distinction matters. Propagating each constant's own CODATA uncertainty through the relevant root gives the deviation in units of that uncertainty:

constant	relative uncertainty	propagated $\delta\alpha^{-1}$	deviation
G	4.6×10^{-5}	2.9×10^{-3}	5.1σ
ℓ_P	2.4×10^{-5}	9.9×10^{-4}	5.1σ
m_P	2.3×10^{-5}	3.0×10^{-3}	5.1σ
k_B	5.7×10^{-7}	4.7×10^{-5}	1441σ

G , ℓ_P and m_P deviate by the same 5.1σ , and they are not independent: ℓ_P and m_P are *defined* through G , so a single poorly determined input carries all three. The common offset places the recommended G high by 2.4 parts in 10^4 : from c , μ_0 and R_∞ the lattice gives $G = 6.67250 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. The combination $h^5/(cG^{13}) = 2^{-25}\pi^{-33}\Omega^{-60}$ ($\theta = 0$) gives the same result without the anchors or α , since a , r and v all cancel: matching it requires $\Delta\Omega/\Omega = +5.1 \times 10^{-5}$ on the CODATA 2014 values (5.1σ), and $+5.9 \times 10^{-5}$ (12σ) with the more precise 2018 and 2022 value, $6.67430(15) \times 10^{-11}$. With c and h exact since 2019, it fixes $G = 6.67250 \times 10^{-11}$, a definite prediction that measurements of G alone can test, and the offset is better read as a prediction about G than as three separate failures. The determinations of G are themselves inconsistent, and CODATA expands their uncertainties by a factor of 3.9 [35]: of the 16 used in 2018 and 2022, five lie within their stated 1σ of the lattice value, among them the corrected JILA pendulum result, $6.67260(25) \times 10^{-11}$ [29], and the atom-interferometer result, $6.67191(99) \times 10^{-11}$ [30], while the six most precise lie 14σ to 25σ above it, though they spread among themselves over 2 parts in 10^4 .

k_B is a different case. Its relative uncertainty is two orders of magnitude smaller than that of G , so its deviation, some 1441σ , cannot be attributed to measurement imprecision. Since 2019 k_B is exact, so primary thermometry, by acoustic gas or Johnson noise methods, now realizes the kelvin rather than measuring k_B , and the 8×10^{-4} gap between the thermal and geometric values cannot be closed by measurement. It is a question for the model. The electromagnetic k_B^{**} of Section 4.3, constructed entirely from electron-sector parameters (g_e , y_e , h , m_e), suggests that the scaling of temperature and entropy at the quantum level is not an independent thermodynamic primitive but has an electromagnetic origin tied to the same lattice; in Article 1a on measuring the CMB, temperature is a function within the radiation domain. Gravity is likewise emergent in this model: Article 3 [5] replaces it with rotating particle–particle orbital pairs at the Planck scale, whose time-averaged sum produces gravitational orbits, so that $G = \ell_P c^2/m_P$, the form of $G^* = V^2 L/M$ in Table 2, is a derived constant rather than a primitive coupling. The two constants outside the precision of the tool, G and k_B , are thus the constants of the two phenomena the model treats as emergent, both measured on macroscopic systems.

Within its scope the lattice has four limits, and they bound what the properties of Section 6.1 can claim. Its predictions disagree with the measurements, for α by 89σ in 2014 and 126σ in 2022 and for h by 11σ to 15σ , and neither offset has moved towards the lattice since 1998. Its stability is that of R_∞ . The line on which the recommended values lie follows from relations the lattice shares with standard physics, so its straightness tests those relations and not Ω . And its reach ends at the macroscopic constants above. The lattice is therefore best described as a parameter-free reference and diagnostic: the properties of Section 6.1 are its method, and the two offsets of Section 6.2 are the open result it exposes.

7 The Algorithmic Argument

This section asks two questions of the relationships established above: were the formulas found by searching a large space of candidates, and do they compress the constants? Both can be answered without reference to whether the geometric interpretation is correct. The first has a sharp answer, given in Section 7.1: the exponents are forced, and the formulas of the metrology tool were not chosen. The second depends on how the rule-set is costed and on what it is compared with, and Section 7.2 answers it conditionally.

7.1 The exponents are not searched for

Any argument of this kind faces one objection above all others: that the expressions were found by trawling a large space of candidates, whose cost the description length omits. Minimum description length charges for a model's *location* in model-space, not only for its length, and if the exponents in $c^4 R_\infty e^3$ were freely chosen the search space would be of order 10^6 expressions per target, some 20 bits each, and no compression could survive it.

The exponents are not freely chosen. Scalar-freedom requires that both r and v cancel, and because $\theta = 8a + 17b$, the unit-number then cancels automatically: *two* constraints, not three. A combination of three constants therefore satisfies a homogeneous 2×3 system, whose integer solutions form a rank-one lattice: the exponents are the unique primitive null vector, determined up to sign and overall scaling. With four constants the same two conditions leave a two-parameter family, which is why four-constant combinations such as the one for h^3 in Section 4.1 are not unique.

Worked example. With scalar content $r^a v^b$,

$$c \rightarrow (0, 1), \quad R_\infty \rightarrow (-9, 5), \quad e \rightarrow (3, -3),$$

the conditions $\sum n_i a_i = 0$ and $\sum n_i b_i = 0$ give

$$-9n_2 + 3n_3 = 0, \quad n_1 + 5n_2 - 3n_3 = 0 \implies (n_1, n_2, n_3) = (4, 1, 3),$$

that is, $c^4 R_\infty e^3$. This is the combination used in Section 6.4. Nothing was selected: given the three constants, the exponents follow, and the verification is a two-line calculation any reader can repeat.

Because the exponents are forced, the only freedom lies in choosing which constants to combine. The constants used in this paper, c , R_∞ , e , h , m_e , λ_e , μ_0 and y_e/g_e , have the following scalar content and unit numbers:

constant	a	b	$\theta = 8a + 17b$
c	0	1	17
R_∞	-9	5	13
e	3	-3	-27
h	13	-5	19
m_e	4	-1	15
λ_e	9	-5	-13
μ_0	7	0	56
y_e/g_e	-1	-2	-42

There are $\binom{8}{3} = 56$ triples, every one of which yields a non-trivial forced exponent vector. Six are degenerate: whenever both R_∞ and λ_e appear, the third constant drops out with exponent zero, because $(-9, 5)$ and $(9, -5)$ cancel as a pair. The six triples therefore collapse onto the single two-constant invariant

$$R_\infty \lambda_e = \frac{\alpha^2}{2},$$

verified against CODATA 2022 to 2×10^{-12} . This is the standard result obtained by combining $R_\infty = \alpha^2 m_e c / 2h$ with $\lambda_e = h / m_e c$; the lattice recovers it as the unique scalar-free pair, which is a check the model could have failed rather than a new relation.

Discounting the degeneracy leaves 51 distinct combinations, so the search freedom is

$$\log_2 51 = 5.7 \text{ bits},$$

Table 13: The forced combinations used in this paper. Each is the unique primitive null vector of its three constants, up to sign; the last row is the degenerate pair. R_1 to R_7 are the dimensionless combinations of Section 4.1, and Pathways A to C are those of Table 9.

constants	exponents	combination	used in
c, R_∞, μ_0	$(35, -7, -9)$	$c^{35} R_\infty^{-7} \mu_0^{-9}$	Eq. (5), R_3^{-1}
c, R_∞, e	$(4, 1, 3)$	$c^4 R_\infty e^3$	Pathway B
c, R_∞, h	$(20, -13, -9)$	$c^{20} R_\infty^{-13} h^{-9}$	Pathway A
R_∞, e, μ_0	$(21, 35, 12)$	$R_\infty^{21} e^{35} \mu_0^{12}$	Pathway C, R_7
c, e, μ_0	$(21, 7, -3)$	$c^{21} e^7 \mu_0^{-3}$	R_1
c, λ_e, μ_0	$(35, 7, -9)$	$c^{35} \lambda_e^7 \mu_0^{-9}$	R_2
c, h, μ_0	$(35, 7, -13)$	$c^{35} h^7 \mu_0^{-13}$	R_4
$c, \mu_0, y_e/g_e$	$(14, 1, 7)$	$c^{14} \mu_0 (y_e/g_e)^7$	R_5
c, m_e, μ_0	$(7, 7, -4)$	$c^7 m_e^7 \mu_0^{-4}$	R_6
R_∞, λ_e	$(1, 1)$	$R_\infty \lambda_e$	the six degenerate triples

against the ~ 124 bits that free exponents would have cost. Table 13 lists the combinations used in this paper; the remaining 41 follow by the same two-line construction.

The trials factor of 5.7 bits is negligible beside every other term in the accounting, and it depends only weakly on the size of the constant set: adding μ_0 and y_e/g_e to six constants raises it from 4.1 to 5.7 bits. The result covers the metrology tool of Section 6 as a whole. The base combination of Eq. (5), Pathways A to C and the seven dimensionless combinations of Section 4.1 are each the forced null vector of their three constants, so none of the tool’s formulas was chosen: once the anchors are fixed, the exponents follow. The only freedom is the choice of anchors, and Section 6.4 shows that the significance of the disagreement with measurement does not depend on it.

7.2 Compression is conditional

Whether the lattice also compresses the constants depends on the baseline it is compared with. Listing the constants it reproduces, to their measured precision, takes about 208 bits. Against that list, with 17 bits for the two scalars, 25 bits for the residuals and 5.7 bits of search, the lattice compresses if its rule-set costs less than about 160 bits. But the list counts constants that standard physics already relates: $\alpha = \mu_0 c e^2 / 2h$, $R_\infty = \alpha^2 m_e c / 2h$, $\lambda_e = h / m_e c$ and $y_e / g_e = e / 4\pi m_e$. Given c , μ_0 and R_∞ , these relations leave two independent quantities, α and h , which take about 58 bits at the CODATA 2014 precision. The lattice reproduces them with residuals of 89σ and 4σ , which cost about 10 bits, and with 5.7 bits of search, so against this baseline it compresses only if its rule-set costs less than about 40 bits. For a rule-set of R bits the compression is therefore about $160 - R$ bits against the list and $42 - R$ bits against standard physics.

Costing the rule-set, which comprises Ω , the θ -algebra, the base-15 lock and the structural choices listed below, is the least rigorous step. A costing of roughly 100 bits is defensible, since the derived expressions chain rather than standing independently: V , L , A and K follow from M , T , P by short rules. That is compression against the list but not against standard physics, and the second is the fair baseline, because the relations it includes are established physics. Compression is therefore claimed here only conditionally: it holds if the rule-set can be stated in under about 40 bits.

What is not charged for. Three structural choices (see Article 1b) carry information without carrying any adjustable number, and a strict accounting should name them even where it cannot cost them precisely.

1. **The definition of Ω .** That $\Omega = \sqrt{\pi e e^{1-e}}$ rather than some other combination of π and e is a choice. It is defended in the companion work on structural grounds, but those grounds are not part of the present accounting.
2. **The capacity postulate and its evaluation point.** The functional $F(x) = e(\pi/x)^x$ is evaluated at $x = e$ rather than at its own extremum $x = \pi/e$. Both the functional and the evaluation point are postulated.
3. **The assignment of π and Ω .** That T carries π and P carries Ω is inherited from prior work rather than derived here.

None of these is a fitted parameter, and none can absorb a numerical discrepancy. But each locates the model within a space of alternatives, and an adversarial accounting would charge for that location. We estimate the three at perhaps 20 to 40 bits combined, which on its own approaches the break-even point of the standard-physics baseline.

7.3 What the argument does and does not establish

It establishes that the formulas were not selected. If a critic grants nothing else, the null-space result of Section 7.1 still stands: the candidate set has 51 members, and their exponents are determined by two linear constraints. That is checkable in a line, it covers every formula of the metrology tool, and it removes the objection that most often defeats numerological claims. It is the strongest single point of the argument.

It establishes two parameter-free predictions. With c , μ_0 and R_∞ as inputs, the lattice reproduces α , e , h , m_e , λ_e and y_e/g_e with no fitted parameter. Standard physics relates four of them to the others (Section 7.2), and the lattice reproduces those relations rather than assuming them, which is a constraint it could have failed. What remains are two independent predictions, α and h , the intercept and slope of Eq. (6). They carry the evidential weight, and on the CODATA 2014 values they differ from measurement by 20 ppb (89σ) and 49 ppb (4σ) (Table 4).

It does not establish compression or the mechanism. Against a baseline that includes standard physics, compression requires a rule-set shorter than about 40 bits (Section 7.2), and the paper does not claim that the rule-set is that short. A short description would in any case be evidence that structure exists, not that this particular structure is the right one: any equally short rule-set reproducing the same constants would have equal claim, and none is currently known.

From forced exponents to physical reality. A sufficiently complex polynomial can fit any dataset. The lattice does not fit a curve to data: it derives the constants from a parameter-free geometric construction whose exponents are fixed by a Diophantine null space, and from c , μ_0 and R_∞ it reproduces six constants to parts in 10^8 (Sections 5.1 and 5.3). That is what a lattice underlying the constants would produce. Whether the constants are the compressed output of such an algorithmic geometry depends on the cost of the rule-set, and whether its two predictions are right is the question that the offsets of Section 6.2 leave open.

8 Conclusions

8.1 Principal results

The analysis rests on results that do not depend on one another; each could hold while the others failed.

A parameter-free reference. From c , μ_0 and R_∞ , with no fitted parameter, the lattice reproduces α , e , h , m_e , λ_e and γ_e to parts in 10^8 (Sections 5.1 and 5.3). Its values are fixed to a few parts in 10^{12} and have stayed fixed across the CODATA adjustments, so the comparison reduces to two numbers (Section 6.1): an offset in α that has stayed at 20 to 25 ppb while its uncertainty fell 24-fold, and an offset in h that appeared between 2006 and 2014 and is nearly 15σ for the value fixed in 2019 (Section 5.3.1). Changing the anchors to c , e and R_∞ moves the disagreement from h to α but leaves its significance unchanged (Section 6.4).

Forced exponents. Every formula used here, the base combination of Eq. (5), Pathways A to C and the seven dimensionless combinations, is the unique null vector of its three constants under scalar cancellation. The candidate set has 51 members, or 5.7 bits of search, so the formulas were not selected (Section 7.1).

Convergence of orthogonal pathways. Pathway A, without e , and Pathway B, without h , agree to 5.9×10^{-10} on CODATA 2022 values, 49 times more closely with each other than either does with the recommended value, with forced exponents and no fitted parameter (Table 9). Because in raw-data terms both depend on the Kibble-balance h (Section 5.4), their agreement shows that the lattice is consistent with c , R_∞ , e and h together, rather than providing two separate confirmations. Pathway C, $R_\infty^{21} e^{35} \mu_0^{12}$, whose exponents are equally forced, agrees with them to 3 ppb and sits slightly apart because μ_0 and e together carry part of the measured α . All three lie 27 to 30 ppb below the recommended value. These results are least vulnerable to the objection that α was tuned, since nothing was tuned.

The permeability as a closure check. In the two-parameter fit, μ_0 enters the objective as one of the six constants, so its reproduction by the fitted (a, r) — $4\pi \times 10^{-7}$ to 1.6×10^{-9} — is a closure check on the fit rather than an out-of-sample prediction. Removing μ_0 from the objective shifts the fitted a by less than 10^{-8} relative and returns μ_0 within a few parts in 10^8 . The residual minimum is unique, as a scan of r over a range 6×10^4 times the working window confirms, and the location of the search window cannot manufacture the result. The definite permeability statement of the model is the parameter-free one: with c , R_∞ , e and h as inputs the lattice gives $\mu_0 = 4\pi \times 10^{-7} (1 + 2.8 \times 10^{-8}) \text{ NA}^{-2}$ (Eq. 7), against the measured $4\pi \times 10^{-7} (1 - 1.3 \times 10^{-10})$ [16]. Since 2019 μ_0 is a measured quantity, so this offset is directly testable.

Collected predictions. Table 14 gathers the falsifiable numbers of the model in one place. All lattice values follow from $\Omega = \sqrt{\pi e^{1-e}}$ with no fitted parameter; where a value depends on the anchor set or the SI frame, the dependence is shown explicitly. The decisive test is the last row: a QED-free determination of α with $u_r < 1 \times 10^{-8}$ separates the lattice value from the recommended value by more than three standard deviations.

8.2 The discrepancy with the recommended value

These results agree on a value of α^{-1} near 137.0359952, against the CODATA 2022 recommendation of 137.035999177(21): a difference of 4.0×10^{-6} , or 2.9×10^{-8} relative, which is 190σ on the recommended uncertainty with the inputs treated as exact. With raw inputs and propagated uncertainties it is 5σ to 23σ for the pathways that use h and about 100σ to 260σ for $(R_\infty, \mu_0 | c)$,

Table 14: Falsifiable predictions of the lattice, with the anchor set or SI frame on which each depends. Offsets are lattice minus recommended, against CODATA 2022 [16] unless stated.

quantity	lattice prediction	anchors / frame	CODATA 2022	
α^{-1}	137.035 996 3688	$c, \mu_0 = 4\pi \times 10^{-7}$ (exact), R_∞	137.035 999 177(21)	
α^{-1}	137.035 995 20	c, e, R_∞ ; no Ω	137.035 999 177(21)	
α^{-1}	137.035 994 894	c, μ_0 (measured), R_∞ ; band-closing	137.035 999 177(21)	
h	$(6.626\,068\,7\text{--}6.626\,069\,7) \times 10^{-34}$ J s	pre-2019 SI, any anchor subset	6.626 070 15 (exact)	−66 t
μ_0	$4\pi \times 10^{-7}(1 + 2.8 \times 10^{-8})$ N A ^{−2}	c, R_∞, e, h	$4\pi \times 10^{-7}(1 - 1.3 \times 10^{-10})$	+
m_e	$9.109\,383\,15 \times 10^{-31}$ kg	c, e, R_∞, Ω	$9.109\,383\,7139(28) \times 10^{-31}$	
G	$6.672\,50 \times 10^{-11}$ m ³ kg ^{−1} s ^{−2}	exact c and h via h^5/cG^{13}	$6.674\,30(15) \times 10^{-11}$	−

decisive test: QED-free α^{-1} (quantum Hall + calculable capacitor), $u_r < 1 \times 10^{-8}$:
lattice 137.035 9952 vs recommended 137.035 9992, a 3σ separation at $u_r = 9.8 \times 10^{-9}$.

15 to 30 times the disagreement between the Rb and Cs measurements, so it is not an artefact of the least-squares adjustment (Section 5.4). In the model’s own terms the misfit is the Ω band of Section 5.2, which contains the geometric Ω and whose range is set by the two offsets of Section 6.2.

The difference has significant implications but is not necessarily indicative of failure. The geometric α can be read as a ‘bare’ lattice value, and the CODATA α as a ‘dressed’ value altered by virtual loops: the most precise readings of the geometric α come from dimensionless combinations of the most precise dimensioned constants, and their only QED content is the bound-state QED in R_∞ , through which a 29 ppb change in α would move the lattice α by only about 10^{-13} . The recommended α , by contrast, is dominated by determinations that use perturbative QED as an intermediary. On this reading a QED-free determination should agree with the lattice. The determinations that exist do not: the quantum Hall values lie on the far side of the recommended value, 2.2σ to 4.3σ from the lattice depending on the pathway, and the atom-recoil values, whose QED enters only through R_∞ and $A_r(e)$, leave little room for an offset (Section 5.4). The existing QED-free data therefore lean against the bare/dressed reading at about 3σ , on measurements made between 1988 and 2001; they do not yet exclude it. The sharpest tension is that the Rb and Cs recoil values agree with each other to 1.2 ppb, 25 times closer than the lattice gap, which on this reading must be explained and currently cannot be. The decisive test, a QED-free determination of α to better than 1×10^{-8} , is set out in Section 6.5.

A key caveat is that the chosen constants must be independent of each other, yet clearly they share units, and so fixing the value of c may influence the readings of the other constants; for example, it defines the metre. Hence multiple combinations need to be referenced and compared.

8.3 Anomaly Detection

The hierarchy identified in Section 6.6, with electromagnetic constants aligning tightly and macroscopic constants diverging, works as a built-in diagnostic. Because the model is a rigid generative structure, once the scalars r, v and the value of a are fixed all remaining predictions are forced, and a future high-precision measurement that moves a constant’s implied Ω into the band of Section 5.2 supports the lattice, while one that moves it out constrains it. The model is thus an anomaly detector for metrology and for itself: a constant whose measured value lies far from the prediction, as the constants of emergent gravity and temperature, G and the thermal k_B , do (Section 6.6), is flagged either as a possible metrological or systematic outlier or as a point where the geometric model must be revised.

8.4 The Electron as the Central Geometric Object

The most physically consequential part of the model is not that its formulas are forced and free of fitted parameters, but that the electron sector appears to be generated by a single dimensionless invariant, ψ (Section 3.5) [1]. Units and scalars cancel in ψ , so the model gives the electron a purely mathematical core, and its dimensional quantities arise only on translation into SI through r and v . The electron is then not an independent particle with separately fitted mass, charge, wavelength, Rydberg linkage and gyromagnetic scale; these are dimensional projections of one geometric construction.

This explains why several high-precision electron constants move together. m_e , λ_e , R_∞ and y_e/g_e are linked through ψ , so their agreement should not be counted as fully independent statistical evidence; their shared residual pattern is itself a diagnostic of whether one invariant can organize the electron-sector constants. The magnetic-monopole decomposition of ψ into three *AL*-type constituents forming a *DDD* structure is at present a model-internal geometric interpretation rather than an established particle-physics result, but it gives the framework a concrete internal mechanism: the electron is constructed from the same base-15 unit-number rules that determine the rest of the lattice.

The electron is therefore the central falsifiable object of the model. The Planck-object construction supplies the geometric stage, the base-15 rule the lattice constraint and Ω the mathematical scale, but ψ is where the framework first becomes a particle model. If future precision measurements preserve the common electron-sector residual structure, the case for a genuine geometric electron strengthens; if they break the ψ -linked pattern, the model will require revision at its most fundamental physical point.

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